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# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



### THESIS

**A JOINT SERVICE OPTIMIZATION  
OF THE  
PHASED THREAT DISTRIBUTION**

by

Brian L. Widdowson

March, 1998

Thesis Advisor:  
Second Reader:

Richard E. Rosenthal  
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19980526 021

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**DTIC QUALITY INSPECTED 2**

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instruction, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188) Washington DC 20503.

1. AGENCY USE ONLY (Leave blank)

2. REPORT DATE  
March 1998

3. REPORT TYPE AND DATES COVERED  
Master's Thesis

4. TITLE AND SUBTITLE

A Joint Service Optimization of the Phased Threat Distribution.

5. FUNDING NUMBERS

6. AUTHOR(S) Brian L. Widdowson

DWAM70262

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Naval Postgraduate School  
Monterey, CA 93943-5000

8. PERFORMING ORGANIZATION  
REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Chairman of the Joint Chiefs of Staff, J-8 (WAD), The Pentagon,  
Washington, D.C.

10. SPONSORING / MONITORING  
AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

12a. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (maximum 200 words).

The decreasing defense budget forces the Department of Defense (DoD) to continually find areas in which to reduce military spending. Traditionally, each service has requested munitions considering itself in isolation. This inevitably leads to excessive munitions acquisition for the DoD as a whole. The Under Secretary of Defense for Acquisition and Technology developed the Capabilities-Based Munitions Requirement (CBMR) process to ensure that the DoD address the munitions acquisition issue from a joint perspective and thus reduce excess.

The CBMR process requires each warfighting CinC to produce a phased threat distribution (PTD). The PTD specifies which friendly platform will be assigned to each enemy platform for a given scenario. This provides the services with estimates of the threats they must be prepared to overcome and the munitions they need. The purpose of this thesis is to help develop the PTD in such a way that the threats are assigned appropriately with limited overlap among the services.

To achieve this purpose, the thesis develops a goal programming model that attempts to find an optimal allocation based on three objectives: minimize friendly casualties, maximize enemy casualties, and maximize adherence to the guidance delineating a proper division of labor among the services.

14. SUBJECT

Phased Threat Distribution, Munition, Capabilities-Based Munition Requirement Process.

15. NUMBER OF  
PAGES

85

16. PRICE CODE

17. SECURITY  
CLASSIFICATION OF  
REPORT

Unclassified

18. SECURITY CLASSIFICATION OF  
THIS PAGE

Unclassified

19. SECURITY CLASSIFICATION  
OF ABSTRACT

Unclassified

20. LIMITATION OF  
ABSTRACT

UL



**Approved for public release; distribution is unlimited**

**A JOINT SERVICE OPTIMIZATION OF THE PHASED THREAT DISTRIBUTION**

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Submitted in partial fulfillment of the  
requirements for the degree of

**MASTER OF SCIENCE IN OPERATIONS RESEARCH**

from the

**NAVAL POSTGRADUATE SCHOOL  
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## **ABSTRACT**

The decreasing defense budget forces the Department of Defense (DoD) to continually find areas in which to reduce military spending. Traditionally, each service has requested munitions considering itself in isolation. This inevitably leads to excessive munitions acquisition for the DoD as a whole. The Under Secretary of Defense for Acquisition and Technology developed the Capabilities-Based Munitions Requirement (CBMR) process to ensure that the DoD address the munitions acquisition issue from a joint perspective and thus reduce excess.

The CBMR process requires each warfighting CinC to produce a phased threat distribution (PTD). The PTD specifies which friendly platform will be assigned to each enemy platform for a given scenario. This provides the services with estimates of the threats they must be prepared to overcome and the munitions they need. The purpose of this thesis is to help develop the PTD in such a way that the threats are assigned appropriately with limited overlap among the services.

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## EXECUTIVE SUMMARY

The decreasing defense budget forces the Department of Defense (DoD) to continually find areas in which to reduce military spending. Traditionally, each service has requested munitions considering itself in isolation. This inevitably leads to excessive munitions acquisition for the DoD as a whole. The Under Secretary of Defense for Acquisition and Technology developed the Capabilities-Based Munitions Requirement (CBMR) Process to ensure that the DoD address the munitions acquisition issue from a joint perspective and thus reduce excess.

The CBMR process requires each warfighting CinC to produce a phased threat distribution (PTD). Department of Defense Instruction 3000.4 defines the PTD as "the CINC's phased assignment of a portion of the enemy's total combat capability (i.e., forces, installations, and organizations) to DOD Component commands. The distribution is a percentage by type of target (e.g., tanks and fighters) by operation plan phases." It simply specifies which friendly platform will be assigned to each enemy platform for a given scenario. This provides the services with estimates of the threats they must be prepared to overcome and the munitions they need. The purpose of this thesis is to help develop the PTD in such a way that the threats are assigned appropriately with limited overlap among the services.

To achieve this purpose, the thesis develops two goal programming models that attempt to find optimal allocations based on three objectives: minimize friendly casualties, maximize enemy casualties, and maximize adherence to the guidance delineating a proper division of labor among the services. First, the thesis describes a monolithic model that optimizes across all phases of a scenario concurrently. The thesis then modifies the

monolithic model to develop a dynamic model. The dynamic model optimizes each phase sequentially, using the results from a prior phase as input for the current.

Both models achieve the objective of providing the Joint Staff with a tool to aid in the PTD development. Although the models still require a considerable effort to acquire data, the requirement is no more extensive than that of past PTDs. The models' most significant benefit is the ease with which the PTD is developed once the data has been collected. The monolithic model solves to optimality on a Sun Ultra 200E workstation with 128 megabytes of memory in less than fifteen minutes and the dynamic model in less than six. The speed with which the models provide solutions allows the analyst to run several scenarios, thus conducting sensitivity analysis. In this manner, the models allow the analyst to formulate improved PTDs.

## ACKNOWLEDGMENTS

I would like to thank several people, without whom this thesis would not have been possible. First, I would like to thank Pete Byrne in the Warfighting Analysis Division of J-8. Pete provided me with an interesting, relevant problem to work on and then was instrumental in solving it. His enthusiasm was contagious and his support unfailing. Second, I would like to thank Professor Richard E. Rosenthal and LtCol Kirk Yost. Their guidance and advice saved me countless hours as I grappled with this problem. Their editing skills are something that you, the reader, will appreciate. These three scholars have made my thesis possible.

I would also like to thank my friends, who have been my "local family" while I have been in Monterey. They have made my stay in Monterey something I will always remember fondly. In particular, I thank my girlfriend, Stephanie Whinery. Her love and support has helped me immeasurably as I worked to complete this thesis.

Finally, my acknowledgements are not complete without a thank you to my parents. Everything that I have accomplished I owe to their continued love and support.



## **I. INTRODUCTION**

### **A. BACKGROUND**

The decreasing defense budget forces the Department of Defense (DoD) to continually find areas in which to reduce military spending. One area that consumes a great deal of money is conventional munitions. Each service has traditionally computed munitions requirements in isolation. This inevitably leads to excessive munitions acquisition for the DoD as a whole. To ensure that the DoD addresses the munitions acquisition issue from a joint perspective, thus reducing excess, the Under Secretary of Defense for Acquisition and Technology developed the Capabilities-Based Munitions Requirement (CBMR) process.

The CBMR process defines a methodology to be used for munitions acquisitions. DoD Instruction (DODI) 3000.4, dated 16 June 1997, tasks the services to use the CBMR process to determine their munitions requirements and ensures that other DoD agencies provide needed documents to the services.

Two documents central to the CBMR process are the Outyear Threat Report (OTR) and the Defense Planning Guidance (DPG). The OTR is the Defense Intelligence Agency's (DIA) estimate of adversary capabilities in the "outyears" (a specified planning horizon)[DODI 3000.4]. The OTR is produced every odd-numbered calendar year and supports the development of the threat distributions produced by the Commanders in Chief of the combatant commands (CinCs). The second document, the DPG, is published by the Secretary of Defense. In addition to requiring the CBMR process, the DPG delineates "national security objectives and policies, the priorities of military missions, and the resource levels projected to be available for the period of time for which such recommendations and proposals are to be effective." [Lovelace and Young, 1995] Included in the DPG are Illustrative Planning Scenarios (IPSS). These scenarios

describe several hypothetical conflicts as a means to highlight some of the anticipated military challenges within the coming years. This provides military planners with common scenarios in which to conduct force analysis and to determine the types of capabilities needed. [Lovelace and Young, 1995]

To satisfy the requirements of the CBMR process (depicted in Figure 1.1), the CinCs (Central Command, U.S. Forces Korea, and Pacific Command) are each required to produce a phased threat distribution (PTD), as shown in the lower left corner of Figure 1.1. The PTD is defined in DODI 3000.4 as

*the CINC's phased assignment of a portion of the enemy's total combat capability (i.e., forces, installations, and organizations) to DOD Component commands. The distribution is a percentage by type of target (e.g., tanks and fighters) by operation plan phases.*

The PTD, based on DIA's OTR and the applicable Operation Plan (OPLAN), allows each service to estimate the threat that it must be prepared to overcome. If each service plans to address all the threats specified in the OTR, then excessive investment in munitions is inevitable. While a small amount of overlap is appropriate, the current goal is to divide the threats among the services in such a way that each service is accountable for a portion that is consistent with its role in the applicable OPLAN.



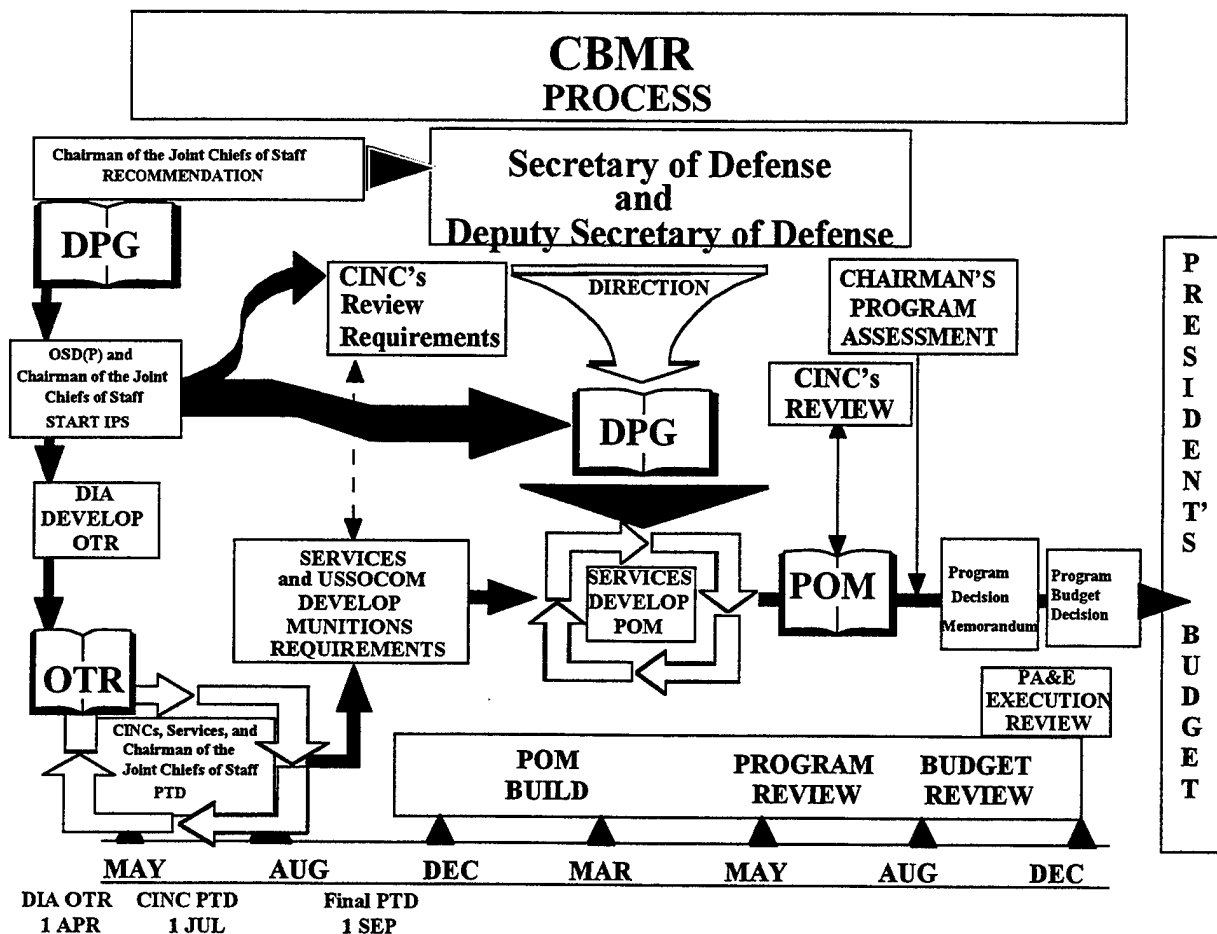


Figure 1.1 The Capabilities-Based Munition Requirements process is used to ensure that excessive overlap of munitions does not exist between the services. This thesis addresses an integral part of this process - the development of the Phased Threat Distribution. The PTD development is depicted in the lower left corner of this graphic [DODI 3000.4].

## B. THE PROBLEM

Developing the PTD is currently a time intensive and laborious process lacking sufficient scientific rigor. The methodology used to form the PTDs varies among the major commands and none make significant use of operations research tools. The purpose of this thesis is to provide the Joint Chiefs of Staff (JCS) with a tool that will efficiently allocate the threat of a given scenario in a defensible manner.

### C. CURRENT PROCEDURE

As stated earlier, the current year PTDs are generated by the warfighting CinCs. The JCS validates these current year PTDs and uses them as input to produce a PTD for the outyears. The PTDs are formed using the DPG (including the IPSs) and the OTR as baseline documents. The JCS provides this PTD to the services, who use it as a basis for their munitions requirements. This process is depicted in Figure 1-2.

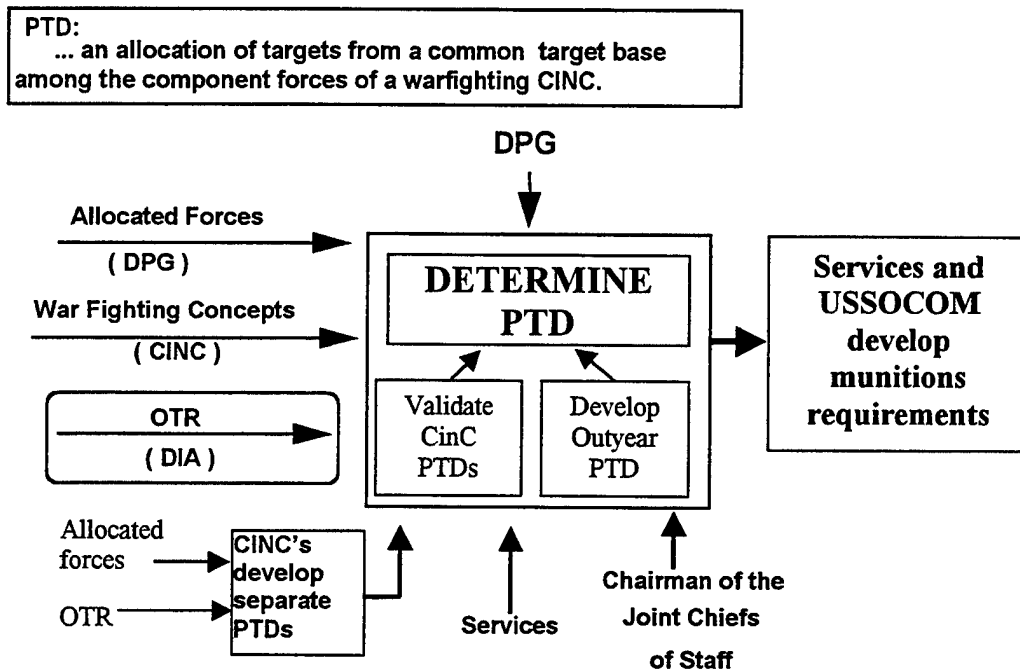


Figure 1.2 The development of the Phased Threat Distribution currently requires the CinCs to each develop a PTD. These PTDs are validated by the Joint Staff and then used as input into a joint PTD for the outyears. This joint PTD allows the services and United States Special Operations Command (USSOCOM) to develop their munitions requirements. [DODI 3000.4]

The method used in the development of the CinCs' PTDs varies with each command. Central Command conducts a time intensive spreadsheet analysis of TACWAR (a deterministic land combat model) [U.S. Army Training and Doctrine Command , 1998] output to develop its PTD. This effort requires the full-time attention of two officers for six months. U.S. Forces Korea (USFK) develops their PTD by making changes to the prior year's PTD as military

judgement deems necessary based on current TACWAR output. The USFK PTD is also used by Pacific Command. Pacific Command does review the USFK PTD, but there is no intensive development method.

#### **D. PROPOSED PROCEDURE**

Upon the completion of the PTD model, the warfighting CinCs will be relieved from the task of producing PTDs. Instead, the Joint Staff will use the model to produce the required PTD without the currently required input of the CinC's PTDs. The CinCs will still be required (and will probably demand the opportunity) to provide input to the Joint Staff for the development of the PTD. This input, however, will be easily provided relative to the task of producing a PTD. The CinCs will provide the concept of operations for their theater and will be involved in reviewing the assumptions and the data sources that the Joint Staff uses. Finally, the CinCs will desire to review the results of the Joint Staff analysis before publication to ensure there are no intolerable points of contention.

In addition to reducing the CinC's burden, the PTD model will have two other benefits. First, it will allow the Joint Staff to achieve the goal of "develop[ing] a common methodology to compute and report quantitative requirements for weapons systems and platforms." [DODI 3000.4] Finally, the relative ease with which the Joint Staff will produce the PTD will allow "what-if" analysis to be conducted on the input data and allocations. The proposed process is depicted in Figure 1.3.

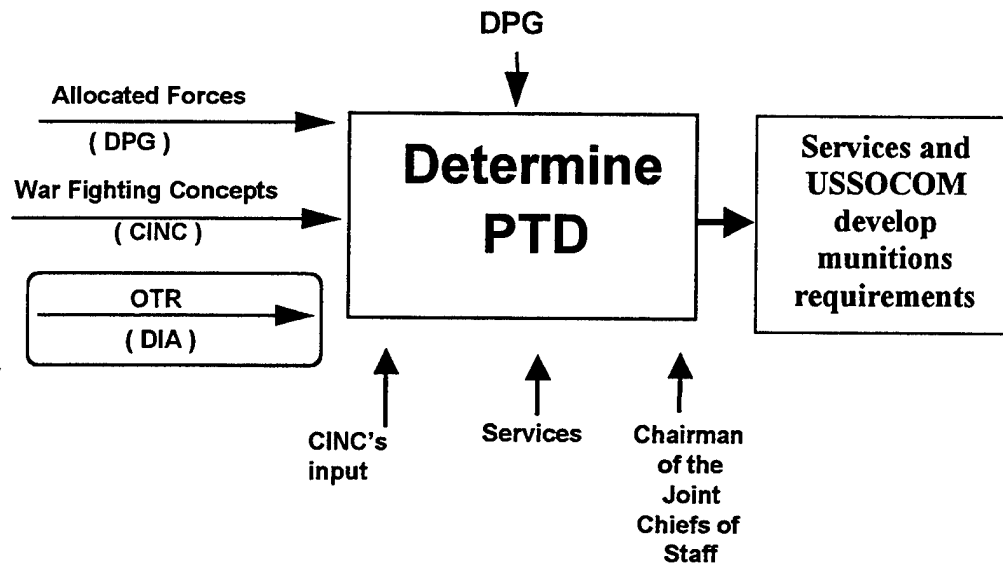


Figure 1.3 With use of the model developed in this thesis, the CinCs will no longer be required to develop PTDs. They will still provide input to the Joint Staff; however, this input will be easily provided relative to the task of producing PTDs. The Joint Staff will be able to produce more timely, beneficial PTDs with reduced effort.

## E. THREAT ALLOCATION

To fully understand how a PTD is formed, the individual elements of the allocation must be understood. In this thesis, each element of the threat allocation is referred to as an assignment. All of the assignments of a scenario collectively form the allocation. It is this allocation that is used to form the PTD. The allocations have a one-to-one correspondence with the PTDs because the PTD is a percentage by type of target by plan phase. A planner may convert the allocation to a unique PTD simply by converting the number of platforms assigned to each service to the corresponding percentage. Figure 1.4 demonstrates a very simplified example of this.

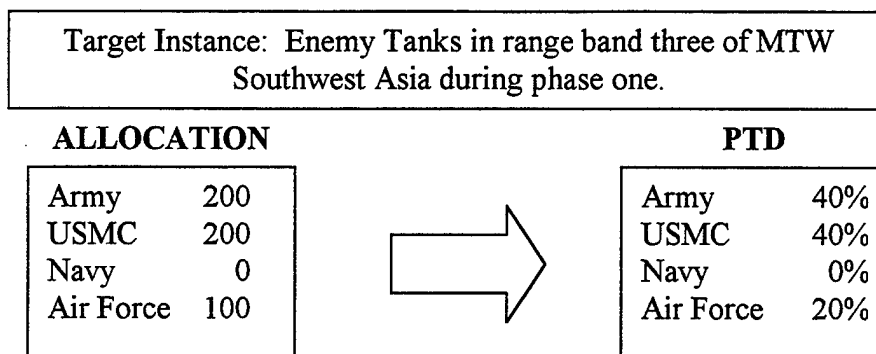


Figure 1.4 The allocation is converted to a unique PTD by computing the percentage of targets allocated for each service.

An optimal allocation, and thus PTD, is valid only for the scenario that is represented by the input data. The following is a generic sample scenario that demonstrates the context within which an allocation is made.

Suppose the United States becomes involved in two major theaters of war (MTW) nearly simultaneously. To protect the interest of the United States, it is necessary to send American service members to both regions. In both regions there is a set of targets that the services, and possibly one ally, must collectively engage. Each service has several platform types and each of these platforms is capable of firing one or more types of munitions. Within each MTW, the targets are spread over the battlefield in five different range bands. These range bands are each 40km in depth, and represent the distance from the forward edge of the battle area (FEBA). Each MTW also has an associated timeline. This timeline divides the conflict into five phases (halt, early build-up, late build-up, early counter-attack, and late counter-attack). These phases may last several hours, days, or weeks.

An allocation for this scenario specifies exactly which service, platform, and munition will engage each target in each MTW, range band, and time period. An example element of this allocation is "The Marines (service) will use tanks (platform) firing sabot rounds (munition) to

attack the BMPs (target) in the second range band of MTW2 during the late counter-attack phase.”

In general, only one platform is assigned to each target. However, a small amount of redundancy is permissible for certain targets. This redundancy is referred to as *overlap* and is intended to “enable the defeat of an unexpected disposition of enemy forces.” [DODI 3000.4] Overlap is accomplished in the model through the use of over-assignments, which are simply assignments that are made on previously assigned targets. The maximum number of over-assignments possible varies by target and is specified in the input data. Assignments and over-assignments are equivalent for the purposes of converting an allocation to a PTD. In contrast to figure 1.4, the sum of assignments and overassignments may exceed 100%.

#### **F. OPTIMALITY CRITERIA**

One must specify the criteria upon which optimality is judged to form an optimal allocation. The following are reasonable goals when determining a threat allocation:

- Maximize target value destroyed
- Maximize the achievement of CinC defined goals by MTW, time phase, or range band.
- Maximize “effectiveness” (a complex criteria not yet defined)
- Minimize unproductive sorties
- Minimize friendly casualties
- Minimize munitions cost

This list is not meant to be exhaustive. Further analysis of the problem by others, with different backgrounds, would undoubtedly find even more.

The previously mentioned objectives can not all be optimized simultaneously. For example, minimizing friendly casualties and maximizing target value destroyed are in conflict.

One approach to this difficulty is to use goal programming [Charnes & Cooper, 1961]. In this approach, the most important criteria are selected, a target value or goal is selected for each criterion, and the under-achievements of the goals are penalized. The model's sponsor considered three goals most important.

### **1. Minimize Friendly Casualties**

First, it is important to minimize friendly casualties. From the planner's point of view, it is also important that enough assets survive to begin the next phase of operations. Placing a goal for each service and MTW on the number of platforms at the start of each time phase incorporates both of these goals. If too many platforms are destroyed during a time phase, the goal is not achieved in the next. This penalizes the overall measure of effectiveness (MOE).

### **2. Maximize Enemy Casualties**

A second goal is to inflict damage upon the enemy. Placing a goal on the military worth of the targets destroyed in each MTW during each phase incorporates this goal. A failure to inflict a given amount of casualties on the enemy again penalizes the final MOE.

### **3. Ensure Services Are Tasked Consistent With Their Capabilities**

Lastly, the model incorporates a service goal for two reasons, *persistence* and *political viability*. The model considers persistence and political considerations by encouraging each service and ally to engage a given "military worth" of enemy in each MTW and phase; failure to do so penalizes the MOE.

#### **a) Persistence**

Persistence is that quality that prevents a model from making large changes to an optimal solution when given a small modification to the input data. It is almost always more desirable to find a nearly optimal solution that is very close to the existing policy than to find a true optimum that requires large changes in current operations [Brown, Dell, and Wood, 1997].

Figure 1-1 makes it obvious that the PTD is one part of a much larger process. For this reason it is desirable to have the model temper changes to the optimal solution caused by minor changes to the inputs. The United States has a significant investment in hardware and force structure to fire the munitions it will procure. This hardware and force structure is clearly designed to make use of particular munitions allocations. Experts can estimate these allocations using previous years' allocations and military judgement. Given that the United States has designed each service to fire certain allocations, it is undesirable to arrive at an optimal solution that is a vast departure from prior years. This "optimal solution" would necessitate an impractical restructuring of the services, which is clearly unacceptable. The service goals allow the model to account for the existing conditions indirectly while still allowing changes that improve on the status quo.

*b) Political viability*

Political viability is another reason service goals are justified within the model. While unpalatable for the modeler, political factors are nevertheless sometimes an important consideration in real world problems. Ignoring these factors is satisfying for the modeler, but can leave the client with a useless product.

Political viability indicates that the involved agencies have confidence in the model. These agencies (the CinCs and Services) have been using PTDs for several years and have much experience in judging allocations. While it may be difficult for these agencies to determine an optimal allocation, they can easily spot a flawed one. Providing service goals effectively steers the model away from these clearly impractical solutions. This prevents the model from losing credibility with the users without forcing a particular solution.



## **G. MEASURE OF EFFECTIVENESS**

These three goals are scaled and a linear combination of the under-achievements and over-achievements of the goals becomes the single MOE for an allocation. The MOE's units are "value of under-achievement". Given this single MOE, the allocation problem can now be formulated as a linear program that seeks to minimize the MOE. This allows the model to properly evaluate the tradeoffs necessitated by conflicting goals. It also provides the user with a tool that can be used to compare different allocations within the same scenario.

Over-achievements are included in the MOE to allow the model to further improve upon solutions that meet all of the goals. The over-achievement terms also allow the model to permit small under-achievements in some goals if they produce very advantageous results in others. All over-achievement terms are scaled with a parameter that is strictly less than one. This scaling ensures the weighting of the underachievement is greater than the weighting of the corresponding overachievement, thus maintaining convexity [Rosenthal, 1983]. This convexity prevents the occurrence of simultaneous under-achievement and over-achievement of the same goal. If an overachievement term had a scale factor greater than one, then the model would be non-convex and it would need a nonlinear constraint to prevent simultaneous under and overachievement. Figure 1.5 (next page) graphically demonstrates this concept.

## **H. CONSIDERATIONS**

As with any issue involving very large sums of money, there are political factors that necessitate several points of discussion. Since the PTD is the basis for munitions requirements, the services will be very sensitive to the results of the JCS PTD. A service that perceives it has been slighted by the JCS PTD will likely attribute, perhaps correctly, the loss of millions of

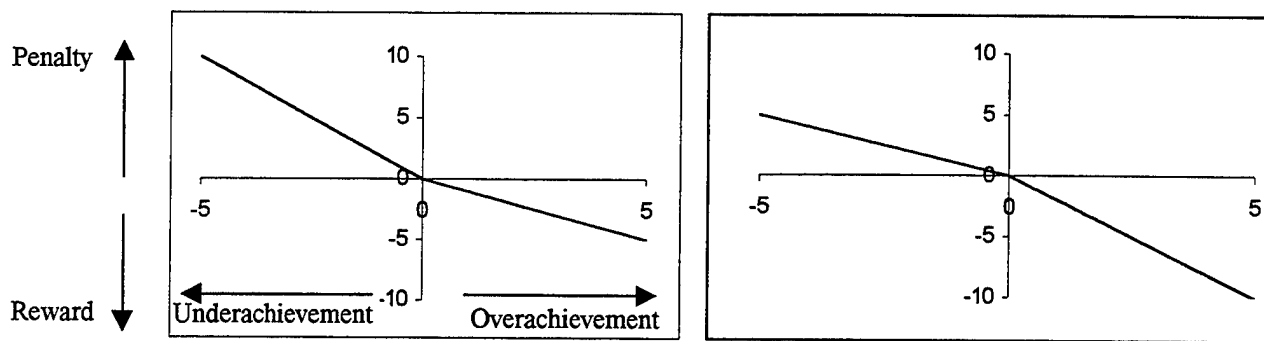


Figure 1.5. The objective function in the left chart rewards overachievement less than it penalizes underachievement, and is thus convex. The objective function in the right chart, however, rewards overachievement more than it penalizes underachievement. It is not convex; therefore, it cannot be minimized with linear programming.

dollars of munitions funding to the JCS distribution. Since the goal of the PTD is to reduce redundancy, it is likely that *each* service, as its munitions acquisition dollars shrink, will feel it has been unjustly slighted. For this reason, it is imperative that the distribution be formulated as objectively as possible. While there may be no reasonable PTD that satisfies everyone, the methodology forming it should be objective and unassailable.

It is also very important to remember that the PTD addresses only the actual warfighting needs of the services. It does not address training ammunition, strategic stockpiles, and those munitions required to ensure that forward-deployed forces are adequately armed. While the PTD will affect munitions acquisitions, the model is not designed as a tool for comparing the platforms used to fire these munitions.

## **II. MODELING APPROACH**

### **A. PROBLEM STATEMENT**

The JCS must "best" allocate the threat of a given scenario among the Navy, Marine Corps, Army, Air Force, and a single allied nation. "Best" refers to the allocation minimizing the degree of failure to achieve the goals of destroying enemy platforms, protecting friendly platforms, and ensuring a reasonable division of labor among the services. This problem is solved in two ways. First, an optimization, referred to as a monolith, which optimizes across all time phases of a scenario concurrently is formulated and solved. To gain additional insight, a dynamic model is also formulated that optimizes phases sequentially, using output from each phase as input for the next.

### **B. MODEL DIFFERENCES**

The monolithic and dynamic models are purposely formulated to be as similar as possible. The inherent difference between the monolithic and dynamic models, however, forces several other differences. The most significant of these differences is the myopic view of the dynamic model. This shortsightedness, unless accounted for, introduces a very undesirable characteristic into the model: greediness. Much less significant, but requiring discussion, is the handling of the platform goal in the dynamic model. The platform goal, as discussed, seeks to ensure that enough platforms exist to begin the *next* phase. This must be modified to be incorporated in the dynamic model, which can only use information pertaining to the current phase.

## **1. Handling Greediness**

A greedy (or myopic) method seeks to improve the objective function despite possible negative effects in an unseen future [Gass and Harris, 1996]. Greediness is not a problem in the monolithic model because the model can consider the later effects of actions taken in an early phase. It is a problem for the dynamic model. This model optimizes each phase individually, so the model cannot assess the future effects of actions taken in a given phase. This myopia causes the model to take any action that improves the objective function, regardless of the resources used in doing so.

Greediness is a problem in a dynamic model that rewards over-achievement. The original purpose of the over-achievement reward was to allow the model to make a trade-off between under-achievement and exceptionally worthy over-achievement elsewhere. The monolithic model scales over-achievements to ensure they are generally preferred less than improving existing under-achievements. This scaling does not help in the dynamic model because once the model has reduced the under-achievements for the current phase, it will waste resources maximally increasing that phase's over-achievements. This makes reducing under-achievements in later phases very difficult.

The dynamic model removes over-achievement rewards from all but the last phase to prevent this waste of resources. This ensures the model does not use resources for a small marginal benefit in an early phase at the great expense of a later phase.

After the dynamic model has run to completion, the analyst may wish to post process the overachievement obtained in the last phase to distribute it throughout the scenario's phases. It seems intuitively obvious that in some cases it is best to make limited overachievements early in a scenario. This is one area in which future work could improve the utility of the model.

## 2. Modifying the Platform Goal

The platform goal of the monolithic model ensures that the number of platforms that begin each phase strives to attain a goal level. Unlike the other goals, the platform goal intrinsically involves more than one phase. Fortunately, another key difference between the models allows the modeler to reconcile the two.

All reinforcements occur *during* the phases in the monolithic model; these same reinforcements occur *between* phases in the dynamic model. The reinforcements in both models are deterministic input data. For this reason, the number of platforms that begin the next phase of the dynamic model is exactly the number of platforms that end the prior phase plus any reinforcements. This allows the modeler to adjust the dynamic goal from beginning the next phase with "X" platforms to ending the *current* phase with "X minus pending reinforcements" platforms. The revised goal now parallels the monolithic goal. For example, assume the monolithic platform goal for tanks in phase three is 200. If the dynamic model will add 100 reinforcement tanks between phases two and three, then the goal of ending phase two with 100 tanks in the dynamic model is equivalent to the goal of beginning phase three with 200 tanks in the monolithic model.

## C. ASSUMPTIONS

Several assumptions are made in the formulation of this model. While the goal is to capture as much of the essence of the real problem as possible, some simplifying assumptions are needed to insure tractability. Another reason for making restrictive assumptions is to ensure usable results.

Professionals who rely on the results of a model must be comfortable that the results reflect reality. While counter-intuitive results are possible (and, when correct, they are a model's

greatest achievement), it is imperative that the analysis provide an explanation consistent with reality. Well-formed assumptions can help prevent unsupportable, counter-intuitive results.

The following assumptions are made in the formulation of this model:

**1. Military Experience and Judgment Have Value.**

The inclusion of service goals provides persistence and incorporates military professionals' expert opinions regarding reasonable divisions of labor between the services. This provides realistic bounds within which the model can find an optimal solution and ensure that the results are organizationally practical and politically feasible.

**2. A Weapon System Has The Same Performance Characteristics Regardless of Which Service Operates It.**

This simplifying assumption is made because available data is not of sufficient fidelity to demonstrate service differences. In addition, any data that portrays a service as less efficient than its sister services will be exploited by the optimization and continually challenged.

**3. Weapons Systems Have Linear Utility in the Short Term.**

It is assumed that each weapon system (friendly and enemy) has a constant value within each phase until a goal is achieved. After the goal has been achieved, the weapon system has a reduced value that will remain constant. This assumption ignores synergistic effects within or between weapon systems. Reducing the value once a goal is achieved incorporates the concept of decreasing marginal returns. If an enemy is already decimated, there is little gained in attacking it further.

**4. Each Phase Lasts Long Enough for Friendly Weapon Systems to Cover the Entire Battlefield.**

This assumption permits friendly weapon systems the full mobility that the model requires. A weapon system may move several hundred kilometers from one time phase to the next. This permits the model to allocate an enemy system to a friendly platform without

accounting for the distance between the systems. Without this assumption the model would be required to consider specific locations of each platform, platform velocities, munitions speeds, etc. The additional factors quickly make the model intractable. Further justifying this assumption, the model uses a controlling set to capture battlefield dynamics. This set also prevents those engagements that would violate physical or tactical considerations. This controlling set, for example, prevents friendly tanks from attacking the enemy's deep rear during a time-phase in which the U.S. is only attempting to halt the enemy.

##### **5. Allied Forces Have Perfect Battlefield Information and Battlefield Dominance.**

It is assumed that the allied forces have complete battlefield knowledge and the ability to select engagements. Complete knowledge is necessary to allow the model to consider all possible allocations. Battlefield dominance prevents the enemy from forcing a disadvantageous engagement, and thus allocation, into the model.

This assumption will tend to favor the use of specialized weapons more than may be warranted. The model always chooses the munition most effective for a given target. In the real world, however, a platform will not always know with certainty what type of target it must engage. Without this information, a real world platform will choose a munition that is generally robust instead of one that is maximally effective for only one of the possible target types.

Although perfect information causes a slight skew in munitions selection, the analyst can anticipate and account for it if necessary. To reduce this problem, the analyst may modify the input data to make the use of a more robust munition more attractive in some cases. In a sense, the modeler can introduce uncertainty by including targets for which non-specialized, robust munitions are the most effective.

## **6. Reinforcement and Resupply are Unaffected by Battles.**

Reinforcement and resupply data for all friendly platforms and munitions are time-indexed. As such, the model assumes that the data accounts for the aggregate effects of battle. The model makes no attempt to adjust reinforcements or resupply based on runtime decisions.

## **D. DATA SOURCES**

Existing data sources provide all of the data used in the model. These data sources are a mixture of senior guidance, outside agency analysis, weapons and environmental data, and simulation outputs. None of the data require a dedicated collection effort, as it has already been obtained for other analyses.

The DPG (including the IPSs) provides the senior guidance data necessary for the model. The OTR, for example, provides data culled from outside agency analysis. Both of these sources have been discussed. Weapons and environmental data are obtained from the input files of existing models. These same models also provide data from their output files and thus warrant discussion.

The Combat Sample Generator (COSAGE) is a division level simulation designed to help planners determine Force Capability and Force Requirements [U.S. Army Concepts Analysis Agency, 1993]. The model is used mainly as a method to calibrate data for theater level models. Input data includes unit organizations, unit strengths, and unit weapons in addition to the actual weapons data. Output files include a killer-victim scoreboard, personnel losses, and ammunition expenditures. The killer-victim scoreboard provides the PTD model with weapons effectiveness data by platform, munition, and target.

The Tactical Warfare Model (TACWAR) is a deterministic ground combat model that examines the interactions of strategic and tactical forces in a given scenario [U.S. Army Training



and Doctrine Command, 1998]. This model is used to help planners develop operational war plans and support operational commands as they conduct exercises or real world operations. Input data again includes unit organization, strength, and weapons; and weapons data. Output can be tailored to meet customer needs. PTD data derived from TACWAR input or output includes the platform and target sets, range band information, munitions types, munitions data, and effectiveness data.

The Weapons Optimization Resource Requirements Model (WORRM) [J8, 1996] is a optimization model that examines only air-to-ground interactions. This model seeks to maximize the military worth of targets destroyed given a cost constraint on the munitions used. WORRM provides the PTD model with the air platforms and the effectiveness data for air munitions against ground targets.



### **III. THE MONOLITHIC MODEL**

#### **A. DISCUSSION**

The monolithic model is a time-indexed linear programming formulation to determine the best allocation when considering all time phases simultaneously. The linear programming model is generated in the General Algebraic Modeling System (GAMS)[Brooke, Kendrick, and Meeraus, 1988] and solved with CPLEX 4.0. The output of the GAMS model consists of the optimal allocation and a measure of its optimality. This optimality figure can then be used to compare the results of runs with differing input data.

A typical scenario generates a matrix with approximately 55,000 rows, 400,000 columns, and 1.5 million non-zero elements. A Sun Workstation model Ultra 200E with 128 megabytes of RAM requires less than ten minutes to solve the model.

#### **B. MONOLITHIC MODEL FORMULATION**

The following is a mathematical description of the monolithic model using the Naval Postgraduate School format. To further aid understanding, additional conventions are followed:

1. All simple indices consist of a single lowercase letter. A single word description follows the index and slashes enclose a sample set.
2. A composite index represents the cross product of two or more sets represented by simple indices. Composite indices are denoted by an uppercase character followed by a lowercase character.
3. A lowercase name indicates given data; relevant indices are subscripted. The units of each type of data are indicated immediately following the name.
4. Uppercase names denote decision variables. Again, the units of each decision variable immediately follow the variable name.

5. In the mathematical formulation, composite indices may be replaced by their component indices to aid understanding of the summations.
6.  $(a,b) \in G(A,B,C)$  indicates that there exist an element  $c \in C$  such that  $(a,b,c) \in G(A,B,C)$ .

## **INDICES**

- s** Service -- /Army, Navy, AirForce, Marine, Allied/  
Indicates instance service. Services are selected based on JCS guidance.
- p** Platform -- /PF1...PFx/  
Indicates the friendly weapons platform instance. Ground weapons platforms are imported from TACWAR; air weapons platforms are from WORRM inputs.
- d** Target types -- /TT1...TTx/  
Indicates instance target type. Target types are based on TACWAR data.
- r** Major Theatre of War (MTW) -- /MTW1...MTW2/  
Model assumes two nearly simultaneous MTWs based on JCS guidance.
- b** Range band -- /RB1...RB5/  
Indicates geographical distance from the FEBA. TACWAR, which provides the data, assumed five range bands with a depth of 40km each.
- m** Munitions Type -- /MT1...MTx/  
Indicates munitions instance. Munitions are imported from TACWAR and WORRM inputs.
- t** phase -- /Halt, E-Build, L-Build, E-Counter, L-Counter/  
Indicates time phase of an MTW. Model assumes the given five phases based on TACWAR.

## **Composite indices**

- Sh<sub>s,p</sub>** Valid shooters  
Set of all possible combinations of service and platform that make up a valid shooter. Thus, the ordered pair (s,p) is an element of set Sh only if service s is equipped with platform p. This set is formed from TACWAR data. It is a sparse subset of all possible pairs (s,p).
- Ta<sub>d,r,b</sub>** Targets  
Set of all possible combinations that constitute a valid target. A valid target instance is a target type that exists in the given MTW and range band. This set is formed from TACWAR data and is also sparse.

$E_{Sh,Ta,m,t}$  Engagement Set  
Set of all possible combinations of (s, p, d, r, b, m, t) that may occur in the model. This set is formed based on JCS guidance and "military common sense."

$O_{eSh,Ta,m,t}$  Over-assignments Engagement Set.  
Set of all possible combinations in which an over-assignment may occur. This set is formed dynamically and is a function of the engagement set, the permitted redundancy, the ekp data, and the service goals.

## **DATA**

### **Target data**

$inittgt_{Ta}$  (targets)  
Initial number of targets.

$tgtgoal_{d,r,t}$  (targets)  
CinC goal for a given target.

$tgtval_{Ta,t}$  (points/target)  
Value of given target under given conditions

### **Platform data**

$initplt_{Sh,r}$  (platforms)  
Initial number of platforms.

$pltgoal_{Sh,r,t}$  (platforms)  
A given platform's availability in phase t.

$pltval_{Sh,r,t}$  (points/platform)  
Penalty weight for platform availability goal

### **Service data**

$svcgoal_{s,d,r,t}$  (percent of targets)  
CinC goal for each service to achieve against a given target under given conditions.

$svcval_{s,d,r,t}$  (points/target)  
Penalty weight for service goal.

### **Operations and Effectiveness data**

$ekp_{Sh,d,r,m,t}$  (targets/platform)  
Expected kills for a specific platform against a specific target under given conditions.

$att_{Sh,d,r,t}$  (platforms/assignment)  
Expected attrition of a specific platform when engaging a specific target under given conditions.

munperasg<sub>d,m</sub> (munitions/target)  
 Required munitions expenditure of type m to kill a target of type d

reinf<sub>sh,r,t</sub> (platforms)  
 Number of friendly reinforcements arriving.

regencap<sub>d,r,t</sub> (targets)  
 Upper bound for the number of targets regenerating during phase t.

regenfr<sub>d,r,t</sub> (unitless)  
 Upper bound for the fraction of dead targets regenerating during time t.

resupply<sub>sh,r,m,t</sub> (munitions)  
 Re-supply of ammunition for a specific platform under given conditions.

over<sub>Ta,t</sub> (unitless)  
 Allowable over-assignments.

#### Scaling data

overparm1 (unitless:  $0 < \text{overparm1} < 1$ )  
 Goal 1 over-achievement reward / under-achievement penalty

overparm2 (unitless:  $0 < \text{overparm2} < 1$ )  
 Goal 2 over-achievement reward / under-achievement penalty

overparm3 (unitless:  $0 < \text{overparm3} < 1$ )  
 Goal 3 over-achievement reward / under-achievement penalty

### **DECISION VARIABLES**

#### Platform

BEGPLT<sub>sh,r,t</sub> (platforms  $\geq 0$ )  
 Number of service s's platforms p in MTW r at start of phase t.

ENDPLT<sub>sh,r</sub> (platforms  $\geq 0$ )  
 Number of service s's platforms p in MTW r at the end of the time horizon.

#### Target

BEGTGT<sub>Ta,t</sub> (targets  $\geq 0$ )  
 Number of live targets of type d in MTW r, range band b, at start of phase t.

ENDTGT<sub>Ta</sub> (targets  $\geq 0$ )  
 Number of live targets of type d in MTW r, range band b, at the end of the time horizon.

### Munitions

$BEGMUN_{Sh,m,r,t}$  (munitions  $\geq 0$ )  
Number of munitions available for assignment in service  $s$ 's platform  $p$  in MTW  $r$ , at start of phase  $t$ .

$ENDMUN_{Sh,m,r}$  (munitions  $\geq 0$ )  
Number of munitions existing at the end of the time horizon.

### Assignments

$ASSIGN_{Es}$  (platforms  $\geq 0$ )  
Number of platforms of type  $p$  from service  $s$  using munition  $m$  assigned to target  $d$  in MTW  $r$  and range band  $b$  during phase  $t$ .

$OVASSIGN_{Oe}$  (platforms  $\geq 0$ )  
Number of platforms of type  $p$  from service  $s$  using munition  $m$  over-assigned to target  $d$  in MTW  $r$  and range band  $b$  during phase  $t$ .

### Regeneration

$REGEN_{Ta,t}$  (targets  $\geq 0$ )  
Number of targets of type  $d$  in MTW  $r$  and range band  $b$  that were dead in phase  $t-1$  and are regenerated for phase  $t$ .

### Achievement

$ACH1O_{Ta,t}$  (targets)  
Over-achievement of target goal

$ACH1U_{Ta,t}$  (targets)  
Under-achievement of target goal

$ACH2O_{Sh,r,t}$  (platforms)  
Over-achievement of platform goal

$ACH2U_{Sh,r,t}$  (platforms)  
Under-achievement of platform goal

$ACH3O_{s,d,r,t}$  (targets)  
Over-achievement of service goal

$ACH3U_{s,d,r,t}$  (targets)  
Under-achievement of service goal

## **FORMULATION**

**GOAL Minimize :**

$$\begin{aligned}
 & \sum_{(d,r) \in Ta} \sum_{t,b} (tgtval_{d,r,b,t} \cdot ACH1U_{d,r,t}) - \\
 & \sum_{(d,r) \in Ta} \sum_{t,r} (overparm1 \cdot tgtval_{d,r,b,t} \cdot ACH1O_{d,r,t}) + \\
 & \sum_{Sh,r,t} (pltval_{Sh,r,t} \cdot ACH2U_{Sh,r,t}) - \\
 & \sum_{Sh,r,t} (overparm2 \cdot pltval_{Sh,r,t} \cdot ACH2O_{Sh,r,t}) + \\
 & \sum_{s,(d,r) \in Ta,t} (svcval_{s,d,r,t} \cdot ACH3U_{s,d,r,t}) - \\
 & \sum_{s,(d,r) \in Ta,t} (overparm3 \cdot svcval_{s,d,r,t} \cdot ACH3O_{s,d,r,t})
 \end{aligned}$$

Minimize under-achievement of the three goals while rewarding over-achievement at a reduced rate.

**Such that:**

### **TARGET GOAL**

$$\begin{aligned}
 & \sum_{\substack{Sh,b,m \text{ s.t.} \\ (Sh,d,r,b,m,t) \in Es}} (ekp_{Sh,d,r,m,t} \text{ASSIGN}_{Sh,d,r,b,m,t}) = tgtgoal_{d,r,t} + ACH1O_{d,r,t} - ACH1U_{d,r,t} \\
 & \forall (d,r) \in Ta, t
 \end{aligned}$$

The targets killed in each MTW and time period differs from the goal by the amount of under or over achievement.

### **PLATFORM GOAL**

$$BEGPLT_{Sh,r,t} = pltgoal_{Sh,r,t} + ACH2O_{Sh,r,t} - ACH2U_{Sh,r,t} \quad \forall Sh, r, t \neq \text{"halt"}$$

The number of platforms of a given type from each service in an MTW differs from that goal by the amount of under or over achievement.



## SERVICE GOAL

$$\sum_{\substack{b \text{ s.t.} \\ (d,r,b) \in Ta}} \left( \sum_{\substack{p,m \text{ s.t.} \\ (s,p,m) \in Es}} (ekp_{s,p,d,r,m,t} \cdot ASSIGN_{s,p,d,r,b,m,t}) + \alpha \cdot \sum_{p,m} (ekp_{s,p,d,r,m,t} \cdot OVASSIGN_{s,p,d,r,b,m,t}) \right) \geq \\ svcgoal_{s,d,r,t} \cdot \sum_b (BEGTGT_{d,r,b,t} + REGEN_{d,r,b,t}) + ACH3O_{s,d,r,t} - ACH3U_{s,d,r,t}$$

$$\forall s,d,r,t \text{ s.t. } svcgoal(s,d,r,t) > 0$$

where  $\alpha = 1$  if overassignment is possible for platform  $(d,r,b,t)$  and 0 otherwise.

The total number of targets killed by each service in an MTW during each phase must exceed the goal plus any over-achievement of that goal or minus any under-achievement of that goal.

## PLATFORM AVAILABILITY

$$\sum_{\substack{d,b,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} ASSIGN_{Sh,d,r,b,m,t} + \sum_{\substack{d,b,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Oe}} OVASSIGN_{Sh,d,r,b,m,t} \leq BEGPLT_{Sh,r,t} \quad \forall Sh,r,t$$

Each platform of each service can only be assigned once in a phase.

## TARGET AVAILABILITY

$$\sum_{\substack{Sh,b,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} ASSIGN_{Sh,Ta,m,t} \leq BEGTGT_{Ta,t} \quad \forall Ta,t$$

Only one platform can be assigned to each target during a phase.

## OVERASSIGNMENT BOUND

$$\sum_{\substack{Sh,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} (ekp_{Sh,d,r,m,t} \cdot OVASSIGN_{Sh,Ta,m,t}) \leq over_{Ta,t} \cdot (BEGTGT_{Ta,t} + REGEN_{Ta,t})$$

$$\forall (d,r,b,t) \in Es$$

Only a given percentage of all targets in each MTW, range band, and phase may be killed by platforms that have been over-assigned.

## REGENERATION BOUNDS

$$\text{REGEN}_{Ta,t} \leq \text{regenfr}_{d,r,t} \cdot (\text{inittgt}_{Ta} - \text{BEGTGT}_{Ta,t}) \quad \forall Ta,t \neq \text{"halt"}$$

$$\text{REGEN}_{Ta,t} \leq \text{regencap}_{d,r,t} \quad \forall Ta,t \neq \text{"halt"}$$

The number of targets that regenerate in a given MTW, range band, and phase is limited by two separate constraints. Regenerations cannot exceed a given fraction of the dead targets nor can they exceed the given limit on the number of targets capable of regenerating.

## TARGET BALANCE

$$\text{BEGTGT}_{Ta,t+1} + \alpha_t \cdot \text{ENDTGT}_{Ta} = \text{BEGTGT}_{Ta,t} + \text{REGEN}_{Ta,t} - \sum_{\substack{Sh,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} (\text{ekp}_{Sh,d,r,m,t} \cdot \text{ASSIGN}_{Sh,Ta,m,t}) \quad \forall Ta,t$$

where  $\alpha_t = 0$  if  $t$  is not the last phase  
 $\alpha_t = 1$  if  $t$  is the last phase.

The number of each target in a MTW, range band, and phase is equal to the number that were there in the prior phase plus any that regenerate minus those that are destroyed.

## PLATFORM BALANCE

$$\begin{aligned} \text{BEGPLT}_{Sh,r,t+1} + \alpha_t \cdot \text{ENDPLT}_{Sh,r} = & \text{BEGPLT}_{Sh,r,t} - \\ & \sum_{\substack{d,b,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} (\text{att}_{Sh,d,r,t} \cdot \text{ASSIGN}_{Sh,d,r,b,m,t}) - \\ & \sum_{\substack{d,b,m \text{ s.t.} \\ (Sh,Ta,m,t) \in Oe}} (\text{att}_{Sh,d,r,t} \cdot \text{OVASSIGN}_{Sh,d,r,b,m,t}) + \\ & \text{reinf}_{Sh,r,t} \quad \forall Sh,r,t \end{aligned}$$

where  $\alpha_t = 0$  if  $t$  is not the last phase  
 $\alpha_t = 1$  if  $t$  is the last phase.

The number of each platform in a MTW from each service during a phase is equal to the number in the prior phase minus those destroyed due to assignments or over-assignments plus any reinforcements that arrive.

# MUNITION BALANCE

$$\begin{aligned}
 \text{BEGMUN}_{Sh,r,m,t+1} + \alpha_t \cdot \text{ENDMUN}_{Sh,r,m} = & \text{BEGMUN}_{Sh,r,m,t} - \\
 & \sum_{\substack{d,b \text{ s.t.} \\ (Sh,Ta,m,t) \in Es}} (\text{ASSIGN}_{Sh,d,r,b,m,t} \cdot \text{munperasg}_{d,m}) - \\
 & \sum_{\substack{d,b \text{ s.t.} \\ (Sh,Ta,m,t) \in Oe}} (\text{OVASSIGN}_{Sh,d,r,b,m,t} \cdot \text{munperasg}_{d,m}) + \\
 & \text{resupply}_{Sh,r,m,t} \\
 & \forall Sh, r, m, t
 \end{aligned}$$

where  $\alpha_t = 0$  if  $t$  is not the last phase  
 $\alpha_t = 1$  if  $t$  is the last phase.

The number of munitions for each platform of a given service in a MTW during a phase is equal to the number in the prior phase minus those expended due to assignments or over-assignments plus any re-supply.



## **IV. THE DYNAMIC MODEL**

### **A. DISCUSSION**

The dynamic model uses the same data set as the monolithic model. Although it inputs all phases concurrently, the dynamic model uses only the data indexed with the current phase in each invocation of the solver. To form the dynamically optimized PTD, the GAMS model is run with the solver placed in a time-indexed loop. For each phase of the problem, the solver produces an optimal allocation for that individual phase. Once that single-phase allocation is optimal, the GAMS code recalculates various parameters for the next run. The dynamic model repeats this loop until all phases are complete.

A typical scenario generates a matrix with approximately 11,000 rows, 75,000 columns, and 325,000 non-zero elements during each phase. A Sun Workstation model Ultra 200E with 128 megabytes of RAM, using CPLEX 4.0, requires approximately five seconds to solve each phase. The entire model, including parameter recalculations, requires approximately five minutes.

Although the dynamic model appears to be only approximately twice as fast on the Sun Workstation, it is many times faster than the monolithic on a personal computer. Test runs indicate that the dynamic model is up to fifteen times faster on these machines. These test runs were conducted on an IBM computer with a Pentium 166 MHz processor and 32 Mb of RAM. The input file for the largest scenario was over 15 Mb. The monolithic model required over fifteen hours to solve this scenario; the dynamic version solved it in slightly less than one hour.

## B. DYNAMIC MODEL FORMULATION

The following is a mathematical description of the dynamic model using the Naval Postgraduate School format. Conventions used are identical to those for the monolithic model.

### INDICES

- s      Service -- /Army, Navy, AirForce, Marine, Allied/  
Indicates instance service. Services are selected based on JCS guidance.
- p      Platform -- /PF1...PFx/  
Indicates the weapons platform instance. Ground weapons platforms are imported from TACWAR; air weapons platforms are from WORRM inputs.
- d      Target types -- /TT1...TTx/  
Indicates instance target type. Target types are based on TACWAR data.
- r      Major Theatre of War (MTW) -- /MTW1...MTW2/  
Model assumes two nearly simultaneous MTWs based on JCS guidance.
- b      Range band -- /RB1...RB5/  
Indicates geographical distance from the FEBA. TACWAR, which provides the data, assumed five range bands with a depth of 40km each.
- m      Munitions Type -- /MT1...MTx/  
Indicates munitions instance. Munitions are imported from TACWAR and WORRM inputs.

### Composite indices

- $Sh_{s,p}$                       Valid shooters  
Set of all possible combinations of service and platform that make up a valid shooter. Thus, the 2-tuple (s,p) is an element of set Sh only if the given service has the given platform. This set is formed from TACWAR data. It will be a sparse subset of all possible combinations of s and p.
- $Ta_{d,r,b}$                       Targets  
Set of all possible combinations that constitute a valid target. A valid target instance is a target type that exists in the given MTW and range band. This set is formed from TACWAR data.
- $ES_{Sh,Ta,m}$                       Engagement Set  
Set of all possible combinations of (s, p, d, r, b, m) that may occur in the model.

$Oe_{Sh,Ta,m}$  Over-assignments Engagement Set.

Set of all possible combinations in which an over-assignment may occur. This set is formed dynamically.

## DATA

### Target data

$inittgt_{Ta}$  (targets)  
Initial number of targets.

$tgtgoal_{d,r}$  (targets)  
CinC goal for a given target.

$tgtval_{Ta}$  (points/target)  
Value of given target under given conditions

### Platform data

$initplt_{Sh,r}$  (platforms)  
Initial number of platforms.

$pltfgoal_{Sh,r}$  (platforms)  
A given platform's availability at the end of the period.

$pltval_{Sh,r}$  (points/platform)  
Penalty weight for platform availability goal

### Service data

$svcgoal_{s,d,r}$  (percent of targets)  
CinC goal for each service to achieve against a given target under given conditions.

$svcval_{s,d,r}$  (points/target)  
Penalty weight for service goal.

### Operations and Effectiveness data

$initmun_{Sh,r,m}$  (munition)  
Munitions of type m for specified service, platform, & MTW triple.

$ekp_{Sh,d,r,m}$  (targets/platform)  
Expected kills for a specific platform against a specific target under given conditions.

$att_{Sh,d,r}$  (platforms/assignment)  
Expected attrition of a specific platform when engaging a specific target under given conditions.

$\text{munperasg}_{d,m}$  (munitions/target)  
Required munitions expenditure of type  $m$  to kill a target of type  $d$

$\text{over}_{Ta}$  (unitless)  
Allowable over-assignment to target  $d$  in MTW  $r$  and range band  $b$ .

#### Scaling data

$\text{overparm1}$  (unitless:  $0 < \text{overparm1} < 1$ )  
Goal 1 over-achievement reward / under-achievement penalty

$\text{overparm2}$  (unitless:  $0 < \text{overparm2} < 1$ )  
Goal 2 over-achievement reward / under-achievement penalty

$\text{overparm3}$  (unitless:  $0 < \text{overparm3} < 1$ )  
Goal 3 over-achievement reward / under-achievement penalty

### **DECISION VARIABLES**

#### Platform

$\text{ENDPLT}_{Sh,r}$  (platforms  $\geq 0$ )  
Number of service  $s$ 's platforms  $p$  in MTW  $r$  at the end of the time horizon.

#### Target

$\text{ENDTGT}_{Ta}$  (targets  $\geq 0$ )  
Number of live targets of type  $d$  in MTW  $r$ , range band  $b$ , at the end of the time horizon.

#### Munitions

$\text{ENDMUN}_{Sh,m,r}$  (munitions  $\geq 0$ )  
Number of munitions existing at the end of the time horizon.

#### Assignments

$\text{ASSIGN}_{Es}$  (platforms  $\geq 0$ )  
Number of platforms of type  $p$  from service  $s$  using munition  $m$  assigned to target  $d$  in MTW  $r$  and range band  $b$ .

$\text{OVASSIGN}_{Oe}$  (platforms  $\geq 0$ )  
Number of platforms of type  $p$  from service  $s$  using munition  $m$  over-assigned to target  $d$  in MTW  $r$  and range band  $b$ .

#### Achievement

$\text{ACHIO}_{Ta}$  (targets)  
Over-achievement of target goal



ACH1U<sub>Ta</sub> (targets)  
Under-achievement of target goal

ACH2O<sub>Sh,r</sub> (platforms)  
Over-achievement of platform goal

ACH2U<sub>Sh,r</sub> (platforms)  
Under-achievement of platform goal

ACH3O<sub>s,d,r</sub> (targets)  
Over-achievement of service goal

ACH3U<sub>s,d,r</sub> (targets)  
Under-achievement of service goal

### FORMULATION

GOAL Minimize :

$$\begin{aligned}
 & \sum_{(d,r) \in Ta} \sum_b (tgtval_{d,r,b} \cdot ACH1U_{d,r}) - \\
 & \sum_{(d,r) \in Ta} \sum_b (overparm1 \cdot tgtval_{d,r,b} \cdot ACH1Q_{d,r}) + \\
 & \sum_{Sh,r} (pltval_{Sh,r} \cdot ACH2U_{Sh,r}) + \\
 & \sum_{s,(d,r) \in Ta} (svcval_{s,d,r} \cdot ACH3U_{s,d,r}) - \\
 & \sum_{s,(d,r) \in Ta} (overparm3 \cdot svcval_{s,d,r} \cdot ACH3O_{s,d,r})
 \end{aligned}$$

Minimize under-achievement of the three goals while rewarding over-achievement at a reduced rate.

Such that:

### TARGET GOAL

$$\sum_{\substack{Sh, b, m \text{ s.t.} \\ (Sh, d, r, b, m, t) \in Es}} (ekp_{Sh, d, r, m} \text{ASSIGN}_{Sh, d, r, b, m}) = \text{tgtgoal}_{d, r} + \text{ACH1O}_{d, r} - \text{ACH1U}_{d, r} \quad \forall (d, r) \in Ta$$

The number of targets killed in each MTW differs from the goal by the amount of under or over achievement.

### PLATFORM GOAL

$$\text{ENDPLT}_{Sh, r} = \text{Nextpltfgoal}_{Sh, r} - \text{Arrivingrein}_{Sh, r} + \text{ACH2O}_{Sh, r} - \text{ACH2U}_{Sh, r} \quad \forall Sh, r$$

The number of platforms of type  $Sh$  to survive in MTW  $r$  differs from the platform goal for the *next* phase minus the reinforcements that will arrive *after* this phase by the amount of under or over achievement.

### SERVICE GOAL

$$\sum_{\substack{b \text{ s.t.} \\ (d, r, b) \in Ta}} \left( \sum_{\substack{p, m \text{ s.t.} \\ (s, p, m) \in Es}} (ekp_{s, p, d, r, m} \cdot \text{ASSIGN}_{s, p, d, r, b, m}) + \alpha \cdot \sum_{p, m} (ekp_{s, p, d, r, m} \cdot \text{OVASSIGN}_{s, p, d, r, b, m}) \right) =$$

$$\text{svcgoal}_{s, d, r} \cdot \sum_b \text{inittgt}_{d, r, b} + \text{ACH3O}_{s, d, r} - \text{ACH3U}_{s, d, r}$$

$$\forall s, d, r \text{ s.t. } \text{svcgoal}(s, d, r) > 0$$

where  $\alpha = 1$  if overassignment is possible for platform  $(d, r, b)$  and 0 otherwise.

The total number of targets killed by each service in an MTW differs from the goal by the amount of over or under achievement.

### PLATFORM AVAILABILITY

$$\sum_{\substack{d, b, m \text{ s.t.} \\ (Sh, Ta, m) \in Es}} \text{ASSIGN}_{Sh, d, r, b, m} + \sum_{\substack{d, b, m \text{ s.t.} \\ (Sh, Ta, m) \in Oe}} \text{OVASSIGN}_{Sh, d, r, b, m} \leq \text{initplt}_{Sh, r} \quad \forall Sh, r$$

Each platform can only be assigned once.

### TARGET AVAILABILITY

$$\sum_{\substack{\text{Sh, m s.t.} \\ (\text{Sh}, \text{Ta}, \text{m}) \in \text{Es}}} \text{ASSIGN}_{\text{Sh}, \text{Ta}, \text{m}} \leq \text{inittgt}_{\text{Ta}} \quad \forall \text{Ta}$$

Only one platform can be assigned to each target.

### OVERASSIGNMENT BOUND

$$\sum_{\substack{\text{Sh, m s.t.} \\ (\text{Sh}, \text{Ta}, \text{m}) \in \text{Es}}} (\text{ekp}_{\text{Sh}, \text{d}, \text{r}, \text{m}} \cdot \text{OVASSIGN}_{\text{Sh}, \text{Ta}, \text{m}}) \leq \text{over}_{\text{Ta}} \cdot \text{inittgt}_{\text{Ta}} \quad \forall (\text{d}, \text{r}, \text{b}) \in \text{Es}$$

Only a given percentage of all targets in each MTW and range band may be killed by platforms that have been over-assigned.

### TARGET BALANCE

$$\text{ENDTGT}_{\text{Ta}} = \text{inittgt}_{\text{Ta}} - \sum_{\substack{\text{Sh, m s.t.} \\ (\text{Sh}, \text{Ta}, \text{m}) \in \text{Es}}} (\text{ekp}_{\text{Sh}, \text{d}, \text{r}, \text{m}} \cdot \text{ASSIGN}_{\text{Sh}, \text{Ta}, \text{m}}) \quad \forall \text{Ta}$$

The final number of each target in a MTW and range band is equal to the number that were there in the beginning minus those that are destroyed.

### PLATFORM BALANCE

$$\begin{aligned} \text{ENDPLT}_{\text{Sh}, \text{r}} &= \text{initpltr}_{\text{Sh}, \text{r}} - \\ &\sum_{\substack{\text{d, b, m s.t.} \\ (\text{Sh}, \text{Ta}, \text{m}) \in \text{Es}}} (\text{att}_{\text{Sh}, \text{d}, \text{r}} \cdot \text{ASSIGN}_{\text{Sh}, \text{d}, \text{r}, \text{b}, \text{m}}) - \\ &\sum_{\substack{\text{d, b, m s.t.} \\ (\text{Sh}, \text{Ta}, \text{m}) \in \text{Oe}}} (\text{att}_{\text{Sh}, \text{d}, \text{r}} \cdot \text{OVASSIGN}_{\text{Sh}, \text{d}, \text{r}, \text{b}, \text{m}}) \quad \forall \text{Sh}, \text{r} \end{aligned}$$

The number of each platform at the end of the period is equal to the number in the beginning minus those destroyed due to assignments or over-assignments.

## MUNITIONS BALANCE

$$\begin{aligned} \text{ENDMUN}_{Sh,r,m} &= \text{initmun}_{Sh,r,m} - \\ &\sum_{\substack{d,b \text{ s.t.} \\ (Sh,Ta,m) \in Es}} (\text{ASSIGN}_{Sh,d,r,b,m} \cdot \text{munperasg}_{d,m}) - \\ &\sum_{\substack{d,b \text{ s.t.} \\ (Sh,Ta,m) \in Oe}} (\text{OVASSIGN}_{Sh,d,r,b,m} \cdot \text{munperasg}_{d,m}) \\ &\quad \forall Sh, r, m \end{aligned}$$

The number of munitions remaining for each platform of a given service in a is equal to the initial number minus those expended due to assignments or over-assignments .

## C. INTERPHASE CALCULATIONS

The interphase calculations merely adjust each phase's data to account for the results of the previous phase. The regeneration computations determine how many enemy platforms have regenerated between the phases. The balance equations modify the data controlling the initial number of platforms, targets, and munitions.

### 1. Regeneration Computations:

$$\text{totdead}(d,r,b,t) = \text{starttgt}(d,r,b,t_1) - \text{ENDTGT.L}(d,r,b)$$

The total number of dead targets is the number that started the first phase minus those that survived the last phase.

$$\text{maxregen}(d,r,b,t) = \text{regenfrf}(d,r,t) * \text{totdead}(d,r,b,t)$$

The maximum number of targets that can regenerate is a fraction of those that have been killed.

$$\text{regen}(d,r,b,t) = \min(\text{regencap}(d,r,t), \text{maxregen}(d,r,b,t))$$

The number of targets that regenerate is either the limit placed on regeneration by the data or a given fraction of the dead targets, whichever is less.

### 2. Balance Computations

$$\text{inittgt}(d,r,b) = \text{ENDTGT.L}(d,r,b) + \text{regen}(d,r,b,t) \quad \forall t \neq \text{late counter-attack}$$

The initial number of targets is equal to the number that survived the prior phase plus those that regenerate.

$$\text{initpltf}(\text{Sh},r) = \text{ENDPLT.L}(\text{Sh},r) + \text{reinf}(\text{Sh},r,t) \quad \forall t \neq \text{late counter-attack}$$

The initial number of platforms is equal to the number that survived the prior phase plus any reinforcements.

$$\text{initmun}(\text{Sh},r,m) = \text{ENDMUN.L}(\text{Sh},r,m) + \text{resupply}(\text{Sh},r,m,t) \quad \forall t \neq \text{late counter-attack}$$

The initial number of munitions is equal to the number available at the end of the prior phase plus any resupply.



## V. RESULTS AND CONCLUSIONS

### A. RUN TYPES

As of publication, the PTD model has been run on only one realistic data set. Despite the *relative* ease with which the PTD is produced, gathering and inputting all the data remains a non-trivial task. The monolithic model, developed first, has been tested on several smaller data sets and the results verified by expert opinion. The realistic data set (classified Top Secret) produced a PTD comparable to those previously developed by the CinCs. Another scenario is currently being readied for input into the PTD model.

The dynamic model has also been run using all of the data sets developed for the monolithic model. As expected, and desired, the dynamic model produces solutions generally comparable to those of the monolithic model.

The monolithic and dynamic models should both be used as tools in the formation of each PTD. Although they are a significant part of the process, it is important to remember that the models are still only tools. Each has strengths and weaknesses that will make it more or less relevant to a particular scenario. While the results of the models will generally be comparable, any wide disparity will require careful analysis. As a starting point, the analyst may wish to look for the prevalence of alternative optima, which may be the cause of differences between results of the two models. One approach to preventing alternative optima is to add an extra, lightly weighted term to the objective function to break ties. The other most likely cause for large differences is the myopic view of the dynamic model. It will be much harder for the analyst to determine the cause for differences attributable to this myopia, however. Unfortunately, no easy solution to this problem is offered in this thesis, but an idea for future research is indicated later in this chapter.

## **B. RESULTS**

The Top Secret classification of the data sets prevent the discussion of results for specific runs. In a general sense, however, the models provide the desired results – a defensible phased threat distribution – according to the project sponsor.

As desired, both models provide the Joint Staff with an optimal allocation for the given input and model parameters. The parallels between the monolithic and dynamic model prevent wildly divergent solutions, yet there are obvious differences. These differences can be attributed to the difference in the overachievement reward during the beginning phases and the myopic view of the dynamic model.

Most importantly, both models provide a PTD that compares favorably with those developed without the model. The distribution provided is optimal with respect to the given scenario and withstands the review of experts.

## **C. BENEFITS OF MODEL**

### **1. Speed**

Both the monolithic and dynamic models significantly reduce the time required to produce a valid PTD. The primary time requirement when using these models is the collection and processing of the input data. Once the analyst prepares the data for the models, producing the PTD occurs relatively quickly. This is in contrast to prior PTDs, where data collection was merely the very beginning of a long process. Additionally, as preparing the input data becomes more automated, using the models will become quicker and easier.

### **2. Reduced Workload**

Although the CinCs will still have input into the CBMR process, the models formulated in this thesis will help eliminate the requirement that they produce the PTD. This obviously



reduces their workload. In addition, however, the Joint Staff will also be able to produce the PTD with less effort than in the past. In addition to providing the PTD, the models provide run reports and sensitivity analysis that allow the analysts to quickly conduct a rough validation of the results. These reports provide an overall picture that allows the analyst to quickly ensure that the answer is reasonable. The models thus automate much of the work associated with validating the PTD.

### **3. Sensitivity Analysis**

As just mentioned, the sensitivity analysis will allow the Joint Staff to more quickly validate a given PTD. In addition, it will allow the analyst insights that may otherwise be unattainable. The models may, for example, highlight a critical munition or expose a service goal that seriously hinders operations. Much of the value in producing the PTD with these models may be in the insights such sensitivity analyses provide.

These models provide the analyst with sensitivity analysis in two ways. First, the model provides a report on the marginal values of the key variables and constraints. This highlights those variables and constraints that are most important in forming a particular PTD. Secondly, the models produce a PTD so quickly that an analyst may run them many times with modified input data. This allows the analyst to determine the effects of specific -- possibly real world -- constraints. For example, the analyst could easily determine the effect on a scenario if a critical munition were to become unavailable. This type of analysis is time-prohibitive without these models.

### **4. More Easily Defended Rationale**

Although having experts determine the PTD based on their experience may be valid, it is very hard to quantify and explain their criteria. This leaves PTDs so developed open to criticism. The models developed in this thesis have easily explained criteria and derive data from generally

accepted sources. Although a critic may still question an individual criterion or data set used in the model, the rationale justifying a single element is much more easily explained and defended than justifying an experts opinion on the PTD in general. For this reason, PTDs produced using these models are much easier to defend from outside agency criticism.

#### **D. SUGGESTED FUTURE WORK**

The sponsor of this thesis very much considers these models works in progress. The models will be continually refined and improved upon into the foreseeable future. The following items may all be fruitfully addressed.

##### **1. Refine the Data**

The most immediately effective improvement to the models is the continued refinement of the data sets. As with any model, improved accuracy within the data set yields improved results. Although cost and time constraints made using existing data sets essential, this clearly forced compromise. The user may wish to modify the model to take advantage of any newly developed data sets that are superior to existing ones. These data sets may be superior in the quality or fidelity of the data provided.

Improved fidelity will also address a current weakness of the models. There are cases in which the models may have a large amount of dual degeneracy which causes alternative optima. This condition is encouraged when the input data sets do not always adequately differentiate between elements within a set. Both Army and Marine tanks, for example, are equally effective against a given target. This equivalence, in some cases, makes the model indifferent between allocating the target between the two platforms. Massive dual degeneracy leaves the model open to criticism.

Improved data sets will help alleviate this problem. Until improved data is available, sensitivity analysis allows one to check for this condition. By modifying the data very slightly, the analyst can discover if dual degeneracy is a problem. For the previous tank example, the analyst could modify the Army's effectiveness by 1%. If this modification caused serious changes to the model's results, then dual degeneracy was present.

## **2. Analytically Reconcile Model Differences**

A possible criticism of having two models aid in the development of the PTD is that they will develop different answers to the same questions. Obviously, they can not both provide *the* best solution. The analyst must recognize that these are only tools, however. Using two different methods to approximate a solution does not invalidate either approximation, it merely gives the analyst more insight. To maximize the value of the two models it would be helpful to develop a procedure to reconcile their results. The largest difference in the models will be due to the myopic view of the dynamic model, which allows overachievement only in the last phase of a scenario. This overachievement must be distributed to bring the models more closely in agreement.

One way to achieve this without destroying the integrity of the dynamic model is to use the results of the dynamic model to fix the assignments for a run of the monolithic model. This would force the monolithic model to make the same assignments as the dynamic model, but it could allocate the overassignments in a more advantageous manner. Another option for spreading the overachievements is to simply divide the overachievements among the phases, analyzing each modification to ensure that it does not invalidate the results of the dynamic model.

### **3. Incorporate Costs**

The models currently lack any information regarding the costs of the munitions that they allocate. One goal of the Joint Staff is to incorporate this cost information and a cost goal. Adding a cost goal to the models will allow the analyst to determine the effects that various fiscal levels have on the PTD. In addition, cost information will allow the model to provide the analyst with the total munitions acquisitions requirement for each service without any additional work.

### **4. Use Loops to Provide Sensitivity Analysis**

Perhaps the easiest modification is to place the models within loops that systematically change various input data. For example, the loop may increase the value of a selected target from 50% to 200% of its original value in 10% increments. Solving the model for each new value provides the analyst with insights otherwise unobtainable.

### **5. Incorporate the Models Within a Simulation**

The dynamic model runs quickly enough that running it within a simulation is a possibility. The simulation would run the dynamic optimization many times using random draws from the appropriate distribution to provide data points for any uncertain data. Each time it runs the optimization, the simulation stores the results. The simulation could then develop a PTD that is most desirable given uncertainty instead of the PTD most desirable given the expected values of uncertain data.

## **E. ACHIEVEMENT OF OBJECTIVE**

### **1. Improving the PTD**

The model produces PTDs that vary slightly from those formulated in the past; thus, there are changes in target and munition allocations. Given the improved development procedure and

increased validation capability, the Joint Staff believes PTDs developed using these models are better than past PTDs. There are obvious changes to the allocations, but how these changes will affect real world operations is, unfortunately, impossible to know. The only way one could determine the effects of various allocations would be to use those allocations in identical, real world scenarios. At some point, simulation may provide a tool with which to measure these differences.

## **2. Reducing the Workload**

Within six months, the CinCs anticipate that they will be relieved of having to produce PTDs. Instead, they will be able to concentrate on providing improved data to the Joint Staff. This data will consist largely of each CinC's concept of operations. The Joint Staff will also have a reduced workload. They will not be required to validate the CinC's PTDs, and will have a tool that will ease the development of the outyear PTD.

## **F. SUMMARY OF RESULTS**

This thesis presents the beginning of a work in progress. We have developed two tools for the Joint Staff to use in developing the PTD. These models produce improved allocations with much less work and they have been validated by comparisons with existing PTDs. They will free the CinCs from the burden of producing PTDs and the services will have better data for planning. Although the tools represent a large improvement over existing procedures, there are still improvements that can be made upon these tools. These improvements have been outlined.



## APPENDIX A - MONOLITHIC GAMS CODE

```
$TITLE    Monolithic Time Phased Threat Distribution - 02 Feb 98
$offsymxref offsymlist offlisting inlinecom { }
$ontext
```

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Monolithic version of the TPTD model.

```
$offtext
```

```
options
```

```
limrow    = 00 { controls number of equations listed per block }
limcol    = 00
solprint  = off { controls whether solver output listed }
lp        = Osl
iterlim   = 150000 { iteration limit }
reslim    = 75000 { solver running time limit in seconds }
;
```

```
SETS
```

```
s      services and allies
p      platform type
d      target type
r      major regional contingency
b      range band
m      munitions
t      time phases
;
```

```
$include c:\data\ptd_a.stt
```

```
SETS
```

```
Sh(s,p)      shooters
Ta(d,r,b)    targets
Es(s,p,d,r,b,m,t) eligible platform-munition-target tuples
;
```

# PARAMETERS

## \* Target data

inittgt(d,r,b) Initial target inventory  
tgtval(d,r,b,t) Target value  
tgtgoal(d,r,t) Desired target destruction

## \* Platform data

initplt(s,p,r) Initial platform inventory  
pltval(s,p,r,t) Penalty weight for platform availability goal  
pltgoal(s,p,r,t) Desired platform availability

## \* Service data

svcval(s,d,r,t) Penalty weight for service goal  
svcgoal(s,d,r,t) Desired target destruction percent by service

ekp(s,p,d,r,m,t) Targets killed per platform  
att(s,p,d,r,t) Platforms attrited per assignment  
reinf(s,p,r,t) Reinforcements of platforms  
resuply(s,p,r,m,t) Resupply of munitions  
regencap(d,r,t) Max target regeneration capacity  
regenfr(c,d,r,t) Max fraction of dead targets that can be regenerated  
munperasg(d,m) Munitions used per assignment or overassignment

## \* Scaling Data

Overparm1 { \ Worth of over-achieving goal in one instance)  
Overparm2 { ---> relative to under-achieving it in another}  
Overparm3 { / MUST BE LESS THAN 1 FOR BOUNDEDNESS.}

## \* Report params

PARAMETER REPORT1(\*,\*,\*,\*,\*,\*,\*,\*)  
PARAMETER TGTREP(\*,\*,\*,\*,\*)  
PARAMETER PLTREP(\*,\*,\*,\*,\*)  
PARAMETER MUNREP(\*,\*,\*,\*,\*,\*)  
PARAMETER SUPMUNREP(\*,\*,\*)  
PARAMETER SVCNUMREP(\*,\*,\*)  
PARAMETER NUMSREP(\*,\*)  
PARAMETER USEDNUMREP(\*)

## \* Report data

startplt(s,p,r,t)  
starttgt(d,r,b,t)  
startmun(s,p,r,m,t)  
finalplt(s,p,r,t)  
finaltgt(d,r,b,t)  
finalmun(s,p,r,m,t)

;

\$include c:\data\ptd\_a.dat

## \* Derived data

parameter

Redund(d,r,b,t) Amount of overassignment where requested

;

Redund(d,r,b,t) = max[ 0, sum(s,svcgoal(s,d,r,t)) - 1 ] ;

set



```

Oe(s,p,d,r,b,m,t) Valid overassignments
;
Oe(s,p,d,r,b,m,t) = yes $(
    Es(s,p,d,r,b,m,t)      { shooter can engage target }
    and Redund(d,r,b,t)     { target candidate for overassign in t }
    and eksp(s,p,d,r,m,t)   { positive eksp }
    and svcgoal(s,d,r,t)    { service goal exists }
)
;

POSITIVE VARIABLES
* Inventory variables
BEGPLT(s,p,r,t)           # of platforms in MTW r at start of t
ENDPLT(s,p,r)             # of platforms existing at end of horizon
BEGTGT(d,r,b,t)           # of live targets existing at start of t
ENDTGT(d,r,b)             # of live targets existing at end of horizon
BEGMUN(s,p,r,m,t)         # of munitions available for assignment in t
ENDMUN(s,p,r,m)           # of munitions existing at end of horizon

* Action variables
ASSIGN(s,p,d,r,b,m,t)     # of platforms assigned to target in t
OVASSIGN(s,p,d,r,b,m,t)  # of platforms overassigned to target in t
REGEN(d,r,b,t)            # of dead targets that regenerate

* Deviation variables
ACH1O(d,r,t)              overachievement of target goal
ACH1U(d,r,t)              underachievement of target goal
ACH2O(s,p,r,t)            overachievement of platform inventory goal
ACH2U(s,p,r,t)            underachievement of platform inventory goal
ACH3U(s,d,r,t)            underachievement of service goal
ACH3O(s,d,r,t)            overachievement of service goal
;

FREE VARIABLE
OBJ
;

* Variable bounds.
BEGTGT.fx(d,r,b,t)$( ord(t) eq 1 ) = inittgt(d,r,b) ;
BEGPLT.fx(Sh,r,t)$( ord(t) eq 1 ) = initplt(Sh,r) ;
BEGMUN.fx(Sh,r,m,t)$( ord(t) eq 1 ) = initmun(Sh,r,m) ;
REGEN.fx(d,r,b,t)$( ord(t) eq 1 ) = 0 ;
REGEN.up(d,r,b,t) = regencap(d,r,t) ;

```

# EQUATIONS

OBJDEF                    define objective  
 ETGTGOAL(d,r,t)        assignment goal per phase across all bs  
 EPLTGOAL(s,p,r,t)     platform availability goal per phase for all bs  
 ESVCGOAL(s,d,r,t)     service goal per tgt per phase across all bs  
 ETGTBAL(d,r,b,t)      target balance equation  
 REGENBND(d,r,b,t)     regeneration bounded by number of dead targets  
 PFAVAIL(s,p,r,t)      assignment bounded by number of platforms avail  
 TGTAVAIL(d,r,b,t)     assignments bounded by number of tgts avail  
 PFBAL(s,p,r,t)        platform balance equation  
 MUNBAL(s,p,r,m,t)     munition balance equation

OBJDEF..

sum( (d,r)\$sum((b),Ta(d,r,b)),  
 sum((t,b),tgtval(d,r,b,t) \* ACH1U(d,r,t)) )  
 - sum( (d,r)\$sum(b,Ta(d,r,b)),  
 sum((t,b),overparm1 \* tgtval(d,r,b,t) \* ACH1O(d,r,t)) )  
 + sum( (Sh,r,t)\$ ( ord(t) gt 1),  
 pltval(Sh,r,t) \* ACH2U(Sh,r,t) )  
 - sum( (Sh,r,t)\$ ( ord(t) gt 1),  
 overparm2 \* pltval(Sh,r,t) \* ACH2O(Sh,r,t) )  
 + sum( (s,d,r,t)\$sum(b,Ta(d,r,b)),  
 svcval(s,d,r,t) \* ACH3U(s,d,r,t) )  
 - sum( (s,d,r,t)\$sum(b,Ta(d,r,b)),  
 overparm3 \* svcval(s,d,r,t) \* ACH3O(s,d,r,t) )

=E=

OBJ

ETGTGOAL(d,r,t)..

sum( (Sh,m,b)\$Es(Sh,d,r,b,m,t),  
 eksp(Sh,d,r,m,t) \* ASSIGN(Sh,d,r,b,m,t))

=E=

tgtgoal(d,r,t)  
 + ACH1O(d,r,t)  
 - ACH1U(d,r,t)

EPLTGOAL(Sh,r,t) \$ ( ord(t) gt 1) ..

BEGPLT(Sh,r,t)

=E=

pltgoal(Sh,r,t) + ACH2O(Sh,r,t) - ACH2U(Sh,r,t)

ESVCGOAL(s,d,r,t) ..

sum( b\$ Ta(d,r,b),  
 sum( (p,m)\$ (Sh(s,p)\$Es(s,p,d,r,b,m,t)),  
 eksp(s,p,d,r,m,t) \* ASSIGN(s,p,d,r,b,m,t))  
 + 1\$Redund(d,r,b,t) \*  
 sum( (p,m)\$ (Sh(s,p)\$Es(s,p,d,r,b,m,t)),  
 eksp(s,p,d,r,m,t) \* OVASSIGN(s,p,d,r,b,m,t) ) )

=E=

svcgoal(s,d,r,t) \* sum( b\$Ta(d,r,b), BEGTGT(d,r,b,t)  
 + REGEN(d,r,b,t) )  
 + ACH3O(s,d,r,t)  
 - ACH3U(s,d,r,t) ;

```

PFAVAIL(Sh,r,t)..
    sum( (d,b,m)$ (Ta(d,r,b) and Es(Sh,d,r,b,m,t)),
        ASSIGN(Sh,d,r,b,m,t) )
+ sum( (d,b,m)$ (Ta(d,r,b) and Oe(Sh,d,r,b,m,t)),
        OVASSIGN(Sh,d,r,b,m,t) )
=L=
    BEGPLT(Sh,r,t)
;

TGTAVAIL(Ta,t)..
    sum( (Sh,m)$Es(Sh,Ta,m,t),
        ASSIGN(Sh,Ta,m,t) )
=L=
    BEGTGT(Ta,t)
;

OVEBND(Ta(d,r,b),t) $(Ta(d,r,b)$Redund(Ta,t)) ..
    sum( (Sh,m)$ (Oe(Sh,Ta,m,t)$Es(Sh,Ta,m,t)),
        ekp(Sh,d,r,m,t) * OVASSIGN(Sh,Ta,m,t) )
=L=
    Redund(Ta,t) * ( BEGTGT(Ta,t) + REGEN(Ta,t) )
;

ETGTBAL(Ta(d,r,b),t)..
    BEGTGT(Ta,t+1)
+ ENDTGT(Ta) $( ord(t) eq card(t) )
=E=
    BEGTGT(Ta,t)
+ REGEN(Ta,t)
- sum( (Sh,m)$ Es(Sh,Ta,m,t),
        ekp(Sh,d,r,m,t) * ASSIGN(Sh,Ta,m,t) )
;

REGENBND(Ta(d,r,b),t) ..
    REGEN(Ta,t)
=L=
    regenfr(c,d,r,t) * ( inittgt(Ta) - BEGTGT(Ta,t) )
;

PFBAL(Sh,r,t)..
    BEGPLT(Sh,r,t+1)
+ ENDPLT(Sh,r) $( ord(t) eq card(t) )
=E=
    BEGPLT(Sh,r,t)
- sum( (d,b,m)$ Es(Sh,d,r,b,m,t),
        att(Sh,d,r,t) * ASSIGN(Sh,d,r,b,m,t) )
- sum( (d,b,m)$ Oe(Sh,d,r,b,m,t),
        att(Sh,d,r,t) * OVASSIGN(Sh,d,r,b,m,t) )
+ reinf(Sh,r,t)
;

```

```

MUNBAL(Sh,r,m,t)..
    BEGMUN(Sh,r,m,t+1)
+ ENDMUN(Sh,r,m) $( ord(t) eq card(t) )
=E=
    BEGMUN(Sh,r,m,t)
- sum((d,b)$ Es(Sh,d,r,b,m,t),
    ASSIGN(Sh,d,r,b,m,t) * munperasg(d,m) )
- sum((d,b)$ (Oe(Sh,d,r,b,m,t)),
    OVASSIGN(Sh,d,r,b,m,t) * munperasg(d,m) )
+ resupply(Sh,r,m,t)
;

model ptd / all / ;
solve ptd using lp minimizing OBJ ;

* Show reports
NUMSREP("PLT",t) = sum ((Sh,r), BEGPLT.L(Sh,r,t));
NUMSREP("TGT",t) = sum (Ta, BEGTGT.L(Ta,t));
NUMSREP("ASS",t) = sum ((Sh,Ta,m), ASSIGN.L(Sh,Ta,m,t));
NUMSREP("OvA",t) = sum ((Sh,Ta,m), OVASSIGN.L(Sh,Ta,m,t));
NUMSREP("MAX",t) = sum ((Sh,Ta(d,r,b),m)$Es(Sh,Ta,m,t),
    min(BEGPLT.L(Sh,r,t),
    BEGTGT.L(Ta,t) ) );

option NUMSREP:0:1:1;
display NUMSREP;

TGTREP (d,r,b,t,"S") = BEGTGT.L(d,r,b,t);
TGTREP (d,r,b,t,"F") = BEGTGT.L(d,r,b,t)
    + REGEN.L(d,r,b,t+1)
    - sum( (Sh,m)$ Es(Sh,d,r,b,m,t),
    exp(Sh,d,r,m,t) *
ASSIGN.L(Sh,d,r,b,m,t) );
option TGTREP:0:3:2;
display TGTREP;

PLTREP (s,p,r,t,"S") = BEGPLT.L(s,p,r,t);
PLTREP (s,p,r,t,"F") = BEGPLT.L(s,p,r,t)
    - sum((d,b,m)$Es(s,p,d,r,b,m,t),
    att(s,p,d,r,t) * ASSIGN.L(s,p,d,r,b,m,t) )
    - sum((d,b,m)$Oe(s,p,d,r,b,m,t),
    att(s,p,d,r,t) * OVASSIGN.L(s,p,d,r,b,m,t) )
    + reinf(s,p,r,t+1);
option PLTREP:0:3:2;
display PLTREP;

USEDMUNREP(m) = sum ((Sh,d,r,b,t),
    ( ASSIGN.L(Sh,d,r,b,m,t)
    +OVASSIGN.L(Sh,d,r,b,m,t) )
    * munperasg(d,m) );
option USEDUNREP:0:0:1;
display USEDUNREP;
MUNREP (s,p,r,m,t,"S") = BEGMUN.L(s,p,r,m,t);
MUNREP (s,p,r,m,t,"F") = BEGMUN.L(s,p,r,m,t)

```

```

- sum(Ta(d,r,b)$ Es(s,p,d,r,b,m,t),
      ASSIGN.L(s,p,d,r,b,m,t) *
      munperasg(d,m) )
- sum( Ta(d,r,b)$ (Oe(s,p,d,r,b,m,t)),
      OVASSIGN.L(s,p,d,r,b,m,t) *
      munperasg(d,m))
      + resuply(s,p,r,m,t+1);

option MUNREP:0:4:2;
display MUNREP;

loop (t,
  SUPMUNREP(s,m,t)$ (ord(t) eq 1) = sum ((p,r),initmun(s,p,r,m));
  SUPMUNREP(s,m,t)$ (ord(t) ne 1) = sum ((p,r),resuply(s,p,r,m,t))
    +SUPMUNREP(s,m,t-1);
);
option SUPMUNREP:0:2:1;
display SUPMUNREP;

loop (t,
  SVMUNREP(s,m,t)$ (ord(t) eq 1) = sum ((p,d,r,b),
    (( ASSIGN.L(s,p,d,r,b,m,t)
      +OVASSIGN.L(s,p,d,r,b,m,t) )
    * munperasg(d,m) ));
  SVMUNREP(s,m,t)$ (ord(t) ne 1) = sum ((p,d,r,b),
    (( ASSIGN.L(s,p,d,r,b,m,t)
      +OVASSIGN.L(s,p,d,r,b,m,t) )
    * munperasg(d,m) ))
    + SVMUNREP(s,m,t-1) ;
);
option SVMUNREP:0:2:1;
display SVMUNREP;

report1 (s,p,d,r,b,m,t,"AS") = ASSIGN.L(s,p,d,r,b,m,t) ;
report1 (s,p,d,r,b,m,t,"OA") = OVASSIGN.L(s,p,d,r,b,m,t) ;
option REPORT1:0:6:2;
display REPORT1;

* THE END *****

```



## APPENDIX B – DYNAMIC GAMS CODE

```
$TITLE   Dynamic Time Phased Threat Distribution - 02 Feb 98
$offsymxref offsymlist offlisting inlinecom { }
$ontext
```

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Dynamic version of the TPTD model.

```
$offtext
```

```
options
  limrow   = 0 { controls number of equations listed per block }
  limcol   = 0
  solprint = off { controls whether solver output listed }
  lp       = osl
  iterlim  = 100000 { iteration limit }
  reslim   = 50000 { solver running time limit in seconds }
;
```

```
SETS
```

```
  s      services and allies
  p      platform type
  d      target type
  r      major regional contingency
  b      range band
  m      munitions
  t      time phases
;
```

```
$include c:\data\ptd_a.stt
```

```
SETS
```

```
  Sh(s,p)          shooters
  Ta(d,r,b)        targets
  Es(s,p,d,r,b,m,t) eligible platform-munition-target tuples
;
```

# PARAMETERS

## \* Target data

inittgt(d,r,b)            Initial target inventory  
tgtval(d,r,b,t)          Target value  
tgtgoal(d,r,t)           Desired target destruction

## \* Platform data

initplt(s,p,r)           Initial platform inventory  
pltval(s,p,r,t)          Penalty weight for platform availability goal  
pltgoal(s,p,r,t)          Desired platform availability

## \* Service data

svcval(s,d,r,t)          Penalty weight for service goal  
svcgoal(s,d,r,t)          Desired target destruction percent by service

## \*Operations and Effectiveness data

initmun(s,p,r,m)          Initial munition inventory  
munperasg(d,m)           munitions fired for each assignment made  
ekp(s,p,d,r,m,t)          Targets killed per platform  
att(s,p,d,r,t)           Platforms attrited per assignment

## \* Scaling Data

Overparm1    {    \                    Worth of over-achieving goal in one instance}  
Overparm2    {                    --->    relative to under-achieving it in another}  
Overparm3    {    /                    MUST BE LESS THAN 1 FOR BOUNDEDNESS.}

## \* Derived data

redund(d,r,b,t)           Amount of overassignment where requested

## \* Report formats

PARAMETER REPORT1(\*,\*,\*,\*,\*,\*,\*,\*)  
PARAMETER TGTREP(\*,\*,\*,\*,\*)  
PARAMETER PLTREP(\*,\*,\*,\*,\*)  
PARAMETER MUNREP(\*,\*,\*,\*,\*,\*)  
PARAMETER DEADREP(\*,\*,\*,\*,\*)  
PARAMETER USEDmunREP(\*)  
PARAMETER SUPmunREP(\*,\*,\*)  
PARAMETER SVMUNREP(\*,\*,\*)  
PARAMETER NUMSREP(\*,\*)  
PARAMETER MARGREP(\*,\*,\*,\*,\*,\*,\*)  
PARAMETER GOALREP(\*,\*,\*,\*,\*,\*)

## \* Inter-phase data

reinf(s,p,r,t)           Reinforcements of platforms  
resupply(s,p,r,m,t)      Resupply of munitions  
regencap(d,r,t)          Max target regeneration capacity  
regenfrc(d,r,t)          Max fraction of dead targets that can be regenerated  
maxregen(d,r,b,t)        # of targets that can regenerate based on dead targets  
regen(d,r,b,t)           #of targets that regenerate: min of maxregen and regencap  
totdead(d,r,b,t)          total targets killed by the end of t



```

* Report data
  startplt(s,p,r,t)           platforms alive at the beginning of each phase
  starttgt (d,r,b,t)           targets alive at the beginning of each phase
  startmun (s,p,r,m,t)         munitions available at the beginning of each phase
  finalplt(s,p,r,t)           platforms alive at the end of each phase
  finaltgt (d,r,b,t)           platforms alive at the end of each phase
  finalmun (s,p,r,m,t)         platforms alive at the end of each phase
  maxmarg                      maximum marginal value of variables
  ass (s,p,d,r,b,m,t)          assignments made each phase
  ovass (s,p,d,r,b,m,t)        over-assignments made each phase
  goallo(d,r,t)                goal 1 over-achievement
  goal2o(s,p,r,t)              goal 2 over-achievement
  goal3o(s,d,r,t)              goal 3 over-achievement
  goal1u(d,r,t)                goal 1 under-achievement
  goal2u(s,p,r,t)              goal 2 under-achievement
  goal3u(s,d,r,t)              goal 3 under-achievement

```

```

;
$include c:\data\ptd_a.dat

```

```

* Dynamic sets

```

```

SET
  tnow(t)                      current phase
  trem(t)                      all phases except the first
  tlast(t)                     last phase
  tprev(t)                     phase one time period before current phase
  tnext(t)                     phase one time period after the current phase
  Oe(s,p,d,r,b,m,t)           valid overassignments

```

```

;
trem(t) = yes $ (ord(t) ne 1);
tlast(t) = yes $ (ord(t) eq card(t));
startplt(Sh,r,t)$ (not trem(t)) = initplt(Sh,r);
starttgt (Ta,t)$ (not trem(t)) = inittgt(Ta);
startmun (Sh,r,m,t)$ (not trem(t)) = initmun(Sh,r,m);
redund(d,r,b,t) = max[ 0, sum(s,svcgoal(s,d,r,t)) - 1 ] ;

```

```

Oe(s,p,d,r,b,m,t) = yes $(
  Es(s,p,d,r,b,m,t)           { shooter can engage target }
  and Redund(d,r,b,t)          { target candidate for overassign in t }
  and ekp(s,p,d,r,m,t)         { positive ekp }
  and svcgoal(s,d,r,t)         { service goal exists }
)
;

```

```

POSITIVE VARIABLES

```

```

* Inventory variables

```

```

  ENDPLT(s,p,r)                # of platforms existing at end of horizon
  ENDTGT(d,r,b)                # of live targets existing at end of horizon
  ENDMUN(s,p,r,m)              # of munitions existing at end of horizon

```

```

* Action variables
  ASSIGN(s,p,d,r,b,m)      # of platforms assigned to target in t
  OVASSIGN(s,p,d,r,b,m)    # of platforms overassigned to target in t

* Deviation variables
  ACH10(d,r)                overachievement of target goal
  ACH1U(d,r)                underachievement of target goal
  ACH2U(s,p,r)              underachievement of platform inventory goal
  ACH20(s,p,r)
  ACH3U(s,d,r)              underachievement of service goal
  ACH30(s,d,r)              overachievement of service goal
;

FREE VARIABLE
  OBJ
;

EQUATIONS
  OBJDEF                    define objective
  ETGTGOAL(d,r,t)           assignment goal per phase across all bs
  EPLTGOAL(s,p,r,t)         platform availability goal per phase across all bs
  ESVCGOAL(s,d,r,t)         service goal per tgt per phase across all bs
  PFAVAIL(s,p,r,t)          assignment bounded by number of platforms available
  TGTAVAIL(d,r,b,t)         assignments bounded by number of tgts avail
  OVEBND(d,r,b,t)           limits overassignments to allowed overlap
  MUNBAL(s,p,r,m,t)         munitions used
  TGTBAL(d,r,b,t)           tgts destroyed
  PFBAL(s,p,r,t)           platforms destroyed
;

** Unlike the monolithic model, the rewards for overachievement have been
** removed except for the last phase. This is to prevent greediness.
** the reward for the last phase of the platform goal is also still not
** applicable, since the original intent was to achieve the proper number
** of platforms to begin the next phase, of which there is none. The only
** overachievement rewards left then are for the terminal munition and service
** goal.

OBJDEF..
  sum( (d,r)$sum(b,Ta(d,r,b)),
  sum((tnow,b),tgtval(d,r,b,tnow) * ACH1U(d,r)) )
  - sum( (d,r)$sum(b,Ta(d,r,b)),
  sum((tnow,b)$tlast(tnow),
      overparm1 * tgtval(d,r,b,tnow) * ACH10(d,r)) )

  + sum( (Sh,r,tnow)$ (not tlast(tnow)),
  pltval(Sh,r,tnow) * ACH2U(Sh,r) )

  + sum( (s,d,r,tnow)$sum(b,Ta(d,r,b)),
  svcval(s,d,r,tnow) * ACH3U(s,d,r) )
  - sum( (s,d,r,tnow)$ (tlast(tnow)$sum(b,Ta(d,r,b))),
  overparm3 * svcval(s,d,r,tnow) * ACH30(s,d,r) )
=E=
  OBJ
;

ETGTGOAL(d,r,tnow)..
  sum( (Sh,b,m)$Es(Sh,d,r,b,m,tnow),

```

```

      eksp(Sh,d,r,m,tnow) * ASSIGN(Sh,d,r,b,m))
=E=
      tgtgoal(d,r,tnow)
      + ACH1O(d,r)
      - ACH1U(d,r)
;

EPLTGOAL(Sh,r,tnow)$(not tlast(tnow)) ..
      ENDPLT(Sh,r)
=E=
      sum (tnext,pltgoal(Sh,r,tnext))
      - reinf (Sh,r,tnow)
      + ACH2O(Sh,r)
      - ACH2U(Sh,r)
;

ESVCGOAL(s,d,r,tnow) ..
      sum( b $Ta(d,r,b),
          sum( (p,m)$Es(s,p,d,r,b,m,tnow),
              eksp(s,p,d,r,m,tnow) * ASSIGN(s,p,d,r,b,m) )
      + 1$Redund(d,r,b,tnow) *
          sum( (p,m)$Oe(s,p,d,r,b,m,tnow),
              eksp(s,p,d,r,m,tnow) * OVASSIGN(s,p,d,r,b,m) ) )
=E=
      svcgoal(s,d,r,tnow) * sum( b$Ta(d,r,b), inittgt(d,r,b) )
      + ACH3O(s,d,r)
      - ACH3U(s,d,r)
;

PFAVAIL(Sh,r,tnow) ..
      sum( (d,b,m)$Es(Sh,d,r,b,m,tnow),
          ASSIGN(Sh,d,r,b,m) )
      + sum( (d,b,m)$Oe(Sh,d,r,b,m,tnow),
          OVASSIGN(Sh,d,r,b,m) )
=L=
      initplt(Sh,r)
;

TGTAVAIL(Ta,tnow) ..
      sum( (Sh,m)$Es(Sh,Ta,m,tnow),
          ASSIGN(Sh,Ta,m) )
=L=
      inittgt(Ta)
;

OVEBND(Ta(d,r,b),tnow)$Redund(Ta,tnow) ..
      sum( (Sh,m)$Oe(Sh,Ta,m,tnow),
          eksp(Sh,d,r,m,tnow) * OVASSIGN(Sh,Ta,m) )
=L=
      Redund(Ta,tnow) * inittgt(Ta)
;

TGTBAL(Ta(d,r,b),tnow) ..
      ENDTGT(Ta)
=E=

```

```

    initttgt(Ta)
- sum( (Sh,m)$Es(Sh,Ta,m,tnow),
      eksp(Sh,d,r,m,tnow) * ASSIGN(Sh,Ta,m) )
;

PFBAL(Sh,r,tnow)..
    ENDPLT(Sh,r)
=E=
    initplt(Sh,r)
- sum((d,b,m)$ Es(Sh,d,r,b,m,tnow),
      att(Sh,d,r,tnow) * ASSIGN(Sh,d,r,b,m) )
- sum((d,b,m)$ Oe(Sh,d,r,b,m,tnow),
      att(Sh,d,r,tnow) * OVASSIGN(Sh,d,r,b,m) )
;

MUNBAL(Sh,r,m,tnow)..
    ENDMUN(Sh,r,m)
=E=
    initmun(Sh,r,m)
- sum((d,b)$Es(Sh,d,r,b,m,tnow),
      ASSIGN(Sh,d,r,b,m) * munperasg(d,m) )
- sum((d,b)$Oe(Sh,d,r,b,m,tnow),
      OVASSIGN(Sh,d,r,b,m) * munperasg(d,m) )
;

model ptd / all / ;

loop (t,

    tprev(t-1) = yes;
    tnow(t) = yes;
    tnext(t+1) = yes;

    solve ptd using lp minimizing OBJ;

* REPORT INFO
    startplt(Sh,r,t)= initplt (Sh,r);
    starttgt (d,r,b,t) = initttgt (d,r,b);
    startmun (Sh,r,m,t) = initmun (Sh,r,m);

    finalplt(Sh,r,t) = ENDPLT.L (Sh,r);
    finaltgt (d,r,b,t) = ENDTGT.L (d,r,b);
    finalmun (Sh,r,m,t) = ENDMUN.L (Sh,r,m);

    ass (Sh,Ta,m,t) = ASSIGN.L(Sh,Ta,m);
    ovass (Sh,Ta,m,t) = OVASSIGN.L(Sh,Ta,m);

    goallo(d,r,t) = ACH1O.L(d,r);
    goal2o(s,p,r,t) = ACH2O.L(s,p,r);
    goal3o(s,d,r,t) = ACH3O.L(s,d,r);

    goal1u(d,r,t) = ACH1U.L(d,r);
    goal2u(s,p,r,t) = ACH2U.L(s,p,r);
    goal3u(s,d,r,t) = ACH3U.L(s,d,r);

```

\* INTERPHASE CALCULATIONS

```

if (trem(t),
    totdead(d,r,b,t) = starttgt(d,r,b,'t1')-ENDTGT.L(d,r,b);
    maxregen(d,r,b,t) = regenfrc(d,r,t) * totdead(d,r,b,t);
    regen(d,r,b,t) = min (regencap(d,r,t),maxregen(d,r,b,t));
)
;

initttgt(d,r,b)$( ord(t) ne card(t)) = ENDTGT.L(d,r,b)
                                         +regen(d,r,b,t)
;

initplt(Sh,r) $( ord(t) ne card(t) ) = ENDTPLT.L(Sh,r)
                                         +reinf(Sh,r,t)
;

initmun(Sh,r,m)$( ord(t) ne card(t) ) = ENDMUN.L(Sh,r,m)
                                         +resupply(Sh,r,m,t)
;

tprev(t-1) = no;
tnow(t) = no;
tnext(t+1) = no;
)
;

```

\* PRINT REPORTS

```

GOALREP("lu","d",d,r,t) = goallu(d,r,t);
GOALREP("lo","d",d,r,t) = goallo(d,r,t);
GOALREP("2u",s,p,r,t) = goal2u(s,p,r,t);
GOALREP("2o",s,p,r,t) = goal2o(s,p,r,t);
GOALREP("3u",s,d,r,t) = goal3u(s,d,r,t);
GOALREP("3o",s,d,r,t) = goal3o(s,d,r,t);

```

```

option GOALREP:0:4:1;
display GOALREP;

```

```

NUMSREP("PLT",t) = sum ((Sh,r),startplt(Sh,r,t));
NUMSREP("TGT",t) = sum (Ta,starttgt(Ta,t));
NUMSREP("ASS",t) = sum ((Sh,Ta,m),ass(Sh,Ta,m,t));
NUMSREP("OvA",t) = sum ((Sh,Ta,m),ovass(Sh,Ta,m,t));
NUMSREP("MAX",t) = sum ((Sh,Ta(d,r,b),m)$Es(Sh,Ta,m,t),
    min(startplt(Sh,r,t),
        starttgt(Ta,t) ) );

```

```

option NUMSREP:0:1:1;
display NUMSREP;

```

```

TGTREP (d,r,b,t,"S") = starttgt(d,r,b,t);
TGTREP (d,r,b,t,"F") = finaltgt(d,r,b,t);
option TGTREP:0:3:2;
display TGTREP;

```

```

PLTREP (s,p,r,t,"S") = startplt(s,p,r,t);
PLTREP (s,p,r,t,"F") = finalplt(s,p,r,t);
option PLTREP:0:3:2;
display PLTREP;

MUNREP (s,p,r,m,t,"S") = startmun(s,p,r,m,t);
MUNREP (s,p,r,m,t,"F") = finalmun(s,p,r,m,t);
option MUNREP:0:4:2;
display MUNREP;

DEADREP (d,r,b,t) = totdead (d,r,b,t);
option DEADREP:0:3:1;
display DEADREP;

USEDMUNREP(m) = sum ((Sh,d,r,b,t),
                    ( ass(Sh,d,r,b,m,t)
                      +ovass(Sh,d,r,b,m,t) )
                    * munperasg(d,m) );
option USEDMUNREP:0:0:1;
display USEDMUNREP;

loop (t,
  SUPMUNREP(s,m,t)$(ord(t) eq 1) = sum ((p,r),initmun(s,p,r,m));
  SUPMUNREP(s,m,t)$(ord(t) ne 1) = sum ((p,r),resupply(s,p,r,m,t))
    +SUPMUNREP(s,m,t-1);
);
option SUPMUNREP:0:2:1;
display SUPMUNREP;

loop (t,
  SVMUNREP(s,m,t)$(ord(t) eq 1) = sum ((p,d,r,b),
    (( ass(s,p,d,r,b,m,t)
      +ovass(s,p,d,r,b,m,t) )
    * munperasg(d,m) ));
  SVMUNREP(s,m,t)$(ord(t) ne 1) = sum ((p,d,r,b),
    (( ass(s,p,d,r,b,m,t)
      +ovass(s,p,d,r,b,m,t) )
    * munperasg(d,m) ))
    + SVMUNREP(s,m,t-1) ;
);
option SVMUNREP:0:2:1;
display SVMUNREP;

report1 (s,p,d,r,b,m,t,"AS") = ass(s,p,d,r,b,m,t) ;
report1 (s,p,d,r,b,m,t,"OA") = ovass(s,p,d,r,b,m,t) ;
option REPORT1:0:6:2;
display REPORT1;

```

```

maxmarg = max ( smax ((Sh,r),ENDPLT.m(Sh,r)),
                smax (Ta,ENDTGT.m(Ta)),
                smax ((Sh,r,m),ENDMUN.m(Sh,r,m)),
                smax ((Sh,Ta,m),ASSIGN.m (Sh,Ta,m)),
                smax ((Sh,Ta,m),OVASSIGN.m (Sh,Ta,m)),
                smax ((d,r),ACH1O.m (d,r)),
                smax ((d,r),ACH1U.m (d,r)),
                smax ((Sh,r),ACH2U.m (Sh,r)),
                smax ((s,d,r),ACH3U.m (s,d,r)),
                smax ((s,d,r),ACH3O.m (s,d,r))
);

MARGREP ("ENDPLT",s,p,r,' ',' ',' ') =
ENDPLT.m(s,p,r)$(ENDPLT.m(s,p,r)>0.9*maxmarg);
MARGREP ("ENDTGT",d,r,b,' ',' ',' ') =
ENDTGT.m(d,r,b)$(ENDTGT.m(d,r,b)>0.9*maxmarg);
MARGREP ("ENDMUN",s,p,r,m,' ',' ') =
ENDMUN.m(s,p,r,m)$(ENDMUN.m(s,p,r,m)>0.9*maxmarg);
MARGREP ("ASSIGN",s,p,d,r,b,m) =
    ASSIGN.m(s,p,d,r,b,m)$(ASSIGN.m(s,p,d,r,b,m)>0.9*maxmarg);
MARGREP ("OVASSIGN",s,p,d,r,b,m) =
    OVASSIGN.m(s,p,d,r,b,m)$(OVASSIGN.m(s,p,d,r,b,m)>0.5*maxmarg);
MARGREP ("ACH1O",d,r,b,' ',' ',' ') =
ACH1O.m(d,r)$(ACH1O.m(d,r)>0.5*maxmarg);
MARGREP ("ACH1U",d,r,b,' ',' ',' ') =
ACH1U.m(d,r)$(ACH1U.m(d,r)>0.5*maxmarg);
MARGREP ("ACH2U",s,p,r,' ',' ',' ') =
ACH2U.m(s,p,r)$(ACH2U.m(s,p,r)>0.5*maxmarg);
MARGREP ("ACH3U",s,d,r,' ',' ',' ') =
ACH3U.m(s,d,r)$(ACH3U.m(s,d,r)>0.5*maxmarg);
MARGREP ("ACH3O",s,d,r,' ',' ',' ') =
ACH3O.m(s,d,r)$(ACH3O.m(s,d,r)>0.5*maxmarg);

option MARGREP:0:1:6;
display MARGREP;

* THE END *****

```





## APPENDIX C – GLOSSARY OF ACRONYMS

CBMR	Capabilities Based Munitions Requirement Process. The methodology required for determination of munitions acquisitions.
CinC	Commander in Chief. For the purpose of this document, refers to the force commander of a major command.
COSAGE	Combat Sample Generator. A division level simulation used primarily to calibrate input data for theater level models.
DIA	Defense Intelligence Agency. Agency responsible for producing the OTR.
DoD	Department of Defense.
DODI	Department of Defense Instruction.
DPG	Defense Planning Guidance. Document produced by the Secretary of Defense requiring the CBMR process.
FEBA	Forward Edge of the Battle Area
GAMS	General Algebraic Modeling System. A commercial software product available for the coding of linear and non-linear models.
IPS	Illustrative Planning Scenario. An important part of the DPG in which hypothetical scenarios are given to aid the services in force planning.
JCS	Joint Chiefs of Staff.
MOE	Measure of Effectiveness.
MTW	Major Theater of War
OPLAN	Operation Plan.
OSD	The Office of the Secretary of Defense.
OTR	Outyear Threat Report. A document produced by DIA to aid the services in force planning.
PA&E	Program Analysis and Evaluation
POM	Program Objective Memorandum.
PTD	Phased Threat Distribution.

TACWAR Tactical Warfare Model. A theater level deterministic combat model.

USFK United States Forces, Korea.

USSOCOM United States Special Operations Command.

WORRM Weapons Optimization Resource Requirements Model. An optimization model that determines the optimal munition choice for air-to-ground interactions.

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